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The effects of radial migration on the vertical structure of Galactic discs

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ABSTRACT

We present evidence that isolated growing discs, subject to internal spiral perturbations, thicken due to both heating *and* radial migration. We show this by demonstrating that the thickness and vertical velocity dispersions of coeval stars depend on their age as well as the change in their radii. While the disc thickens due to internal processes, we find that this induces only a minor amount of flaring. We further demonstrate the consequences of such thickening on the structural properties of stellar populations and find that they qualitatively agree with recent studies of the Milky Way disc.

Key words: Galaxy: disc – Galaxy: evolution – galaxies: evolution – galaxies: kinematics and dynamics – galaxies: spiral.

1 INTRODUCTION

Stars born at one radius within a disc galaxy such as the Milky Way (MW) need not remain at that radius indefinitely, even if they retain nearly circular orbits. Sellwood & Binney (2002) have shown that the interaction of stars with transient spirals at their corotation resonance (CR) can lead to substantial changes in angular momentum of the stars without significant heating. This radial migration has many important consequences for properties of stellar populations, such as giving rise to a flattened age–metallicity relation with substantial scatter (Sellwood & Binney 2002; Roškar et al. 2008b; Schönrich & Binney 2009a).

While it is typically assumed that the vertical and in-plane motions of stars are largely decoupled, it is becoming increasingly apparent that some coupling exists and that it affects the detailed chemodynamical structure of discs. For example, as stars migrate outwards they experience a smaller vertical restoring force and are therefore expected to reach greater height above the mid-plane. Schönrich & Binney (2009a,b) explored this idea by using analytical chemical evolution models that allowed stars to radially migrate and assumed that in the process of migration their velocity dispersions were conserved. They showed that under these assumptions it was possible to create a thick disc entirely from migrated stars that shared many of the chemodynamical properties of the thick disc in the MW.

Disc thickening due to migrating stars has also been studied in simulations by Loebman et al. (2011), who showed that outwards migrating stars end up in the more vertically extended component of a double sech^2 profile density fit. Similar to Schönrich & Binney (2009b), they found that subdividing the populations by either kinematics or chemical abundances gives rise to an artificial thin/thick disc dichotomy. Thus, radial migration appears as yet another possible mechanism for creating a thickened disc component, but because it relies entirely on internal processes it is conceptually orthogonal to most other thick-disc formation scenarios.

Other mechanisms for creating a thickened disc component involve either heating the disc through bombardment (e.g. Quinn, Hernquist & Fullagar 1993), forming a thickened disc from the remnants of a gas-rich last major merger (Brook et al. 2004) or accreting an extra-planar stellar component (Abadi et al. 2003). All three of these mechanisms are a result of the cosmological environment and consequently the thick disc is often considered to be an indicator of the MW's cosmic history. On the other hand, a thickened component built up by migration has little to do with cosmological evolution but depends largely on internal disc dynamics. If migration can influence the properties of the thicker stellar component, then it can muddle the signatures of cosmologically relevant events that undoubtedly marked the early stages of the MW's disc formation. The formation of the thick disc through heating by clump instabilities is in principle also an in situ formation mechanism (Bournaud, Elmegreen & Martig 2009), but it still relies on the cosmological environment for the rapid gas accretion.

Sales et al. (2009) proposed using the eccentricity distribution of thick-disc stars to distinguish between thick-disc formation mechanisms. Consequently, several groups sought to apply the Sales et al. (2009) eccentricity test to observational data. Dierickx et al. (2010), Casetti-Dinescu et al. (2011) and Wilson et al. (2011), all reached a general conclusion that the eccentricity distribution is mostly inconsistent with the accretion scenario, but broadly consistent with the other three (the gas-rich merger scenario being the most favourable). The eccentricity distribution is not alone enough to break the degeneracies among the models, but such constraints become stronger once metallicities and abundances are also considered: Liu & van de Ven (2012) showed that low-eccentricity stars from a SEGUE survey (Yanny et al. 2009) sample follow a continuous distribution

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in the [Fe/H]–[α/Fe] plane while the high-eccentricity stars seem disconnected in this plane.

The separation in the [α/Fe] versus [Fe/H] plane of thick- and thin-disc selected stars lends strong support to the idea that the two discs are formed as separate structures (Bensby, Feltzing & Lundström 2003). In such a formation scenario, the variation of structural parameters in [α/Fe] versus [Fe/H] for two distinct populations would be expected to be abrupt even for an unbiased sample of stars. However, recently Bovy et al. (2012a) found that such a strong dichotomy does not exist for a selection function-corrected sample of SEGUE stars. Bovy et al. (2012a) find a smooth distribution of structural trends when stars are separated into mono-abundance populations with alpha-rich, metal-poor large scaleheight populations having a short scalelength, while metal-rich stars have low scaleheights and long scalelengths (a similar result was found for the high-α SEGUE population by Cheng et al. 2012). These findings are consistent with Bensby et al. (2011) who also found a short scalelength for targeted thick-disc stars using entirely different data. These recent results contrast strongly with the usual view that the thick disc has a larger scaleheight and a longer scalelength, as is obtained, for example, by fitting the stellar density with a two-component model (Jurić et al. 2008).

The stars in individual abundance bins are also vertically isothermal (Bovy et al. 2012b), strengthening the argument that they are single populations. Both of these properties seem to favour strong internal evolution for the MW disc, because if a disc builds up entirely from internal processes, a distinct second component that dominates away from the plane does not readily form. It is, however, also possible to recover many of these trends in the cosmological context if the disc initially forms hot and thick, subsequently forming increasingly thinner populations as its self-gravity gradually increases (Bird et al. 2013; Stinson et al. 2013).

However, even if stars form in a thicker component early in the history of the disc, they do not become immune to disc perturbations. *N*-body simulations have shown that stars in the thick disc were also able to migrate though to a lesser extent (Solway, Sellwood & Schönrich 2012). The same study showed that migrating stars on average (but not individually) conserve their vertical action, rather than their vertical energy.

Recently, Schönrich & Binney (2012) studied the co-dependence of vertical and horizontal motions analytically. They found that the vertical motion significantly affects the detailed in-plane velocity distributions. Following Binney (2010), the coupling between the two components of motion was achieved through the assumption of vertical action invariance. This assumption has been shown to be a reasonable approximation by more rigorous torus modelling (Binney & McMillan 2011). While the above analytic studies have focused on the co-dependence of oscillations about the guiding centre and the vertical motion, in this paper we explore the implications of radial migration (i.e. the change in the guiding centres) for the vertical distribution. Under action conservation in a radially increasing disc potential, any change in radius will lead to a modification of the vertical motion. However, while stars oscillate in radius by up to ∼1–2 kpc they might migrate radially by many kpc and we can therefore expect that migration would have a larger effect on their vertical motion.

However, Minchev et al. (2012b) argue that vertical action conservation *prevents* the stars from thickening as they migrate outwards. They support this claim with results from isolated *N*-body+SPH simulations from the GalMer data base (Chilingarian et al. 2010) as well as sticky-particle semi-cosmological simulations (Martig et al. 2012). Their conclusion disagrees sharply with the findings of Schönrich & Binney (2009b), who assumed *energy* conservation rather than action conservation when modelling the vertical distribution in their analytic models. However, it also disagrees with Loebman et al. (2011) who used high-resolution *N*-body/SPH simulations and found that the migrated population formed a thicker component that was shown to be broadly consistent with MW thick-disc trends in both chemistry and kinematics.

In Roškar et al. (2012), we focused on the details of the radial migration mechanism in a suite of *N*-body/SPH simulations. We found evidence that the largest migrations were taking place at the CR of dominant spirals, confirming the mechanism proposed by Sellwood & Binney (2002). In this paper, we elucidate the connection between radial migration and vertical disc structure using the fiducial simulation of Roškar et al. (2012). To gain insight into the physical nature of the processes involved, we compare the simulation data with expectations based on the simplified analytic models of Schönrich & Binney (2012). The paper is organized as follows: our methods are presented in Section 2; the basic results are presented in Section 3; we compare the results to a simple analytic model for disc thickening in Section 4; we discuss the consequences of such thickening on the trends in structural properties of stellar populations in Section 5.

2 SIMULATIONS

We analyse the same *N*-body/SPH simulation that we used in Loebman et al. (2011). This simulation is a re-run of the fiducial simulation analysed in Roškar et al. (2008a,b, 2012) using metal diffusion, but is otherwise identical in every respect. The initial conditions consist of two NFW (Navarro, Frenk & White 1997) haloes, one of dark matter with a mass of $10^{12}\,M_{\odot}$ and the other of gas in hydrostatic equilibrium with a mass of $10^{11}\,M_{\odot}$, both sampled with 10^6 particles. The gas also rotates with $\lambda = 0.039$ and $j \propto R$, where j is the specific angular momentum and R is the cylindrical radius. We use the code GASOLINE (Wadsley, Stadel & Quinn 2004) to perform the computation. Once the simulation begins, the gas cools and collapses into a disc that forms stars according to a standard prescription (Stinson et al. 2006) with a density threshold of 0.1 amu cc^{-1} and a temperature cutoff of 1.5×10^4 K. Star particles provide 'feedback' to the gas by injecting it with energy and polluting it with supernova (SN) ejecta. The simulations are evolved completely in isolation and no cosmological context is included. By the end of the simulation, the disc is populated by more than 2×10^6 star particles. The softening we use for the baryonic component is 50 pc. The reader is referred to Roškar et al. (2008a) and Roškar et al. (2012) for further details on the simulations. The simulations were analysed with the aid of the PYTHON-based analysis packages PYNBODY (Pontzen et al. 2013)[1] and IPYTHON (Pérez & Granger 2007).

Our simulation code includes prescriptions for the generation of metals in Type Ia and II supernova explosions as well as in asymptotic giant branch stars. SN II yields are taken from (Raiteri, Villata & Navarro 1996), SN Ia yields from Thielemann, Nomoto & Yokoi (1986) and the mass returned to the interstellar medium (ISM) via stellar winds follows Weidemann (1987). We can therefore follow the abundances of α elements relative to Fe (oxygen is used as the α element proxy). Due to uncertainties in metal yields, the absolute values of [O/Fe] do not match with observations, but relative trends are nevertheless informative. For details on the metal enrichment implementation see Stinson et al. (2006). As in Loebman et al.

[1] http://code.google.com/p/pynbody/

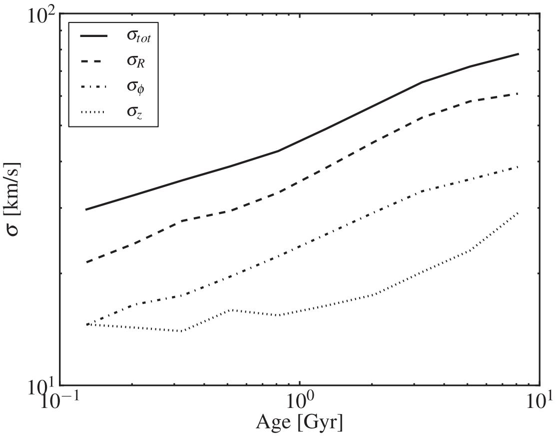

Figure 1. Age–velocity dispersion plot for stars with $7.5 < R$[kpc] < 8.5 and $|z| < 200$[pc] after 10 Gyr of evolution.

(2011), the simulation we use here also implements a prescription for the diffusion of metals through the ISM.

3 RESULTS

The simulation yields a disc that is kinematically similar to that of the MW. In Fig. 1, we show the age–velocity dispersion relation for stars that are found in the 'solar neighbourhood' at the end of the simulation. We define solar neighbourhood as a disc region between 7.5–8.5 kpc and within 200 pc from the plane. The disc scalelength in our model is ∼2.5 kpc, so this is a reasonable approximation to the region of the MW disc where analogous relations have been observed. The age–velocity relation from the Geneva–Copenhagen survey (Holmberg, Nordström & Andersen 2009) shows similar power-law dependences and values, though in our case the oldest stars are hotter by ∼10 per cent. The vertical velocity dispersion shows a shallow dependence for the young stars, similar to that shown for the GCS sample (Seabroke & Gilmore 2007). Part of the increased dispersion for the old stars may be due to the fact that our simulation yields a slightly more massive disc (the circular velocity $V_c \sim 250\,\mathrm{km\,s^{-1}}$). Fitting power laws to the relations in Fig. 1, we obtain power-law indices of 0.24, 0.26, 0.25 and 0.15 for $\sigma_{\rm tot}$, σ_R, σ_ϕ and σ_z respectively. These indices are lower than those observed in the MW, indicating that an additional source of heating may be needed. On the other hand, the fact that the heating indices are not too high also means that the amplitudes of non-axisymmetric structure forming in the disc are not unreasonably high. A related issue is that the youngest stars are not quite as kinematically cool as the observations due to the dispersion floor in the gas component, which also contributes to higher overall velocity dispersions. We discuss this last point further in Section 4.

3.1 Thickening and vertical velocity dispersion due to migration and heating

Because radially migrating stars feel a smaller restoring force towards the mid-plane as they move outwards, the amplitude of their vertical oscillations increases. However, at the same time, heating processes due to the recurring spiral structure also heat the stars, regardless of whether they have migrated or not, providing another source of increased vertical motion. Comparison between figs 3 and 5 in Loebman et al. (2011) shows this implicitly. Their fig. 3 shows that when significant migration is present, the stars dominating the distribution away from the plane have migrated from the interior of the disc. By contrast, in the model with less migration shown in their fig. 5, the populations away from the plane are there mostly because of heating (i.e. they are old) rather than migration (they formed in situ).

In Fig. 2, we show the relative dependence of stellar population thickness on radial migration and heating. As a measure of thickness, we use the model-independent $z_{\rm rms} \equiv \sqrt{\bar{z}^2}$. Each panel shows the distribution of $\Delta z_{\rm rms} = z_{\rm rms,now} - z_{\rm rms,form}$, i.e. the change in thickness since birth, as a function of age and $\Delta R \equiv R_{\rm now} - R_{\rm form}$, for a given range of formation radii. The distributions shown in each of the panels are made by selecting particles in a particular range of *formation* radii (indicated at the top of each panel). We bin the particles in this space on a grid – each bin then corresponds to roughly a coeval population (born at approximately the same radius, of the same age and migrated by the same amount). The contours show the mass density of particles.

If all the changes to the thickness of each population were due to heating, we would expect age to be the determining factor in setting the $z_{\rm rms}$. As a result, the 2D gradient in $\Delta z_{\rm rms}$ would be predominantly in the vertical direction. Conversely, if all the changes in thickness were due to radial migration, the gradient would be horizontal.

The gradients in all of the panels of Fig. 2 are *diagonal*, with $\Delta z_{\rm rms}$ increasing in the direction of older ages and larger ΔR, implying that radial migration as well as heating contribute substantially to the vertical thickness of the stellar populations. This is true for all ages and all ΔR. For positive ΔR, $\Delta z_{\rm rms}$ is a monotonic function of ΔR. Stars migrating inwards decrease their $z_{\rm rms}$ so long as they stay away from the central bulge region. Those stars that migrate to the very centre of the disc increase their $z_{\rm rms}$, presumably due to rapid heating that occurs there due to the presence of multiple inner Lindblad resonances and a weak oval that develops in the centre from time to time. The high velocity dispersions for these stars indicate that this may indeed be the case (see Fig. 3 discussed below). The oldest stars in the interior of the disc (upper-left corners of the top two panels) are on eccentric orbits and therefore not as affected by disc perturbations but their $z_{\rm rms}$ decreases due to adiabatic contraction. Our main findings from Fig. 2 are that (1) at any given age, thickness is a monotonic function of ΔR, and (2) at any ΔR, thickness is a monotonic function of age.

We find a very similar kind of co-dependence for σ_z, the vertical velocity dispersion, shown in Fig. 3. The velocity dispersions of the stars migrating outwards are *lower* than the dispersions of those migrating inwards. This is to be expected because the stars are moving to a region of a shalower mid-plane potential and because stars in the inner disc heat very efficiently (due to inner Lindblad resonances or non-axisymmetric structure). Nevertheless, the dispersion clearly depends on both parameters, age as well as distance migrated since birth, consistent with Fig. 2.

In Figs 2 and 3, we have shown that the thickness and vertical velocity dispersion of coeval stellar populations depend on the magnitude of migration as well as their age. In other words, at a fixed present-day radius, the velocity dispersion and thickness of a population depends on its birth location *and* its time of formation. However, we have also shown that σ_z of stars actually *decreases* as they migrate outwards, so can these stars entering the solar neighbourhood from the inner disc then still masquerade as the thick disc? From Fig. 3, we can see that the old (>8 Gyr) stars that have migrated outwards by several kpc have vertical velocity dispersions of 40–50 km s^{-1}. These stars comprise the α-old population that in the MW has very similar velocity dispersions (Bovy et al. 2012b; Liu

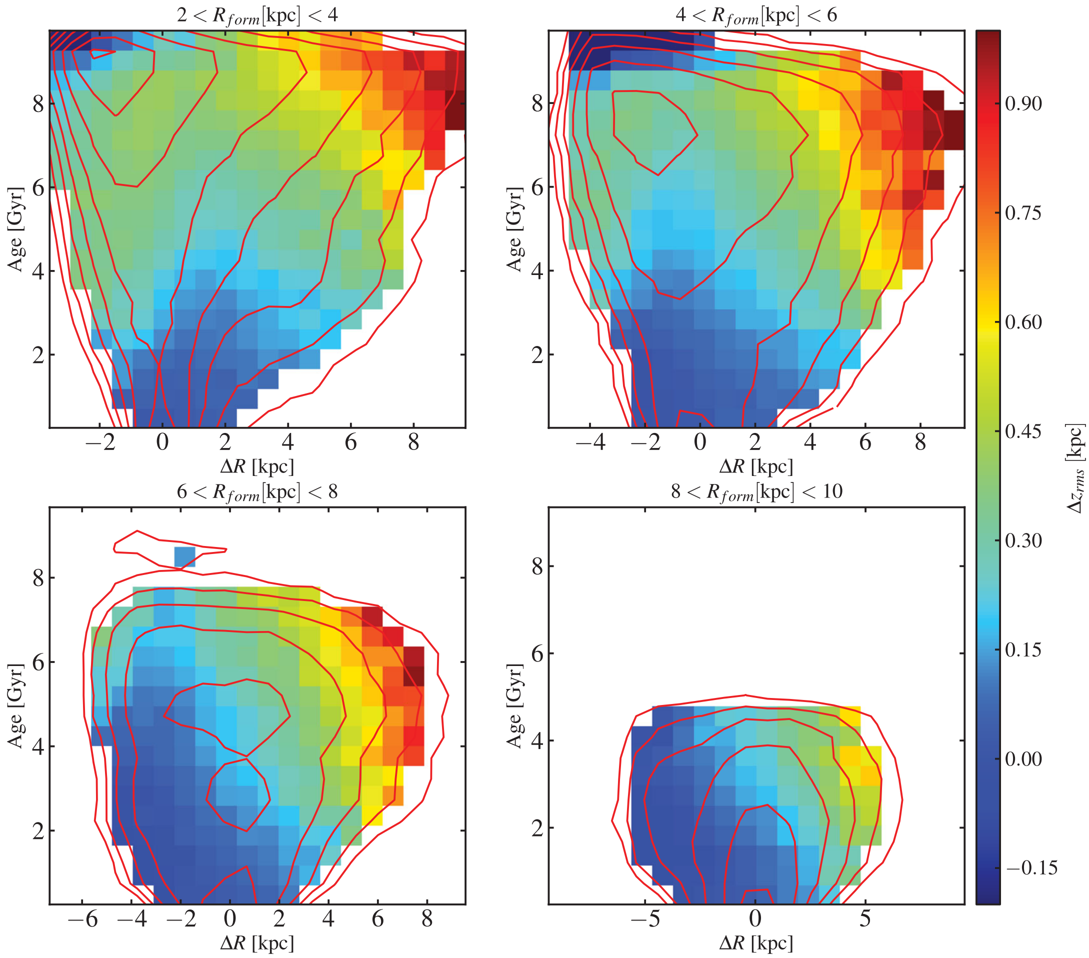

Figure 2. Mean change in thickness as a function of age and ΔR plotted in four different bins of formation radii. Contours show the particle mass density and are logarithmically spaced from 10 to 10^4 particles per bin. Colours correspond to the change in thickness, defined as the root mean square of the vertical displacement from the mid-plane, $z_{\rm rms} = \sqrt{\bar{z}^2}$. Increase in thickness is a function of both, age and ΔR, i.e. stars undergo heating but their vertical distribution is also affected by the effective change in potential as they migrate inwards or outwards in the disc. Note that the *x*-axis range is different in each panel.

& van de Ven 2012); note that for the purposes of *defining* the thick disc, the 'characteristic' σ_z is taken to be $\sim$35 km s^{-1} (Bensby et al. 2003), a criterion met by this migrated population. Even if the populations 'cool' as they migrate outwards, their present-day properties can remain consistent with a thickened, α-rich population. Note that in our models the only stars with the appropriate σ_z are >5 Gyr old and certainly the only ones that are *thickened* enough to represent the thick disc are even older and have migrated substantially. The in situ young stars have $\sigma_z < 20$ km s^{-1}, making up the 'thin' disc.

Minchev et al. (2012b) similarly found that velocity dispersions decrease as stars migrate outwards. They interpreted the decrease in σ_z as an indication that radial migration cannot contribute to a thickened population. However, because the stars migrating from the inner to outer disc begin with a relatively high velocity dispersion, they still end up somewhat kinematically hotter relative to the in situ population when they arrive at the outer disc. Furthermore, as Fig. 2 clearly shows, the outward migrating populations thicken vertically as well. We find therefore that it is indeed possible to form a thickened and kinematically hotter component with the aid of radial migration. Our results therefore appear qualitatively similar to those of Minchev et al. (2012b), although we arrive at different conclusions. We speculate further that possible discrepancies between our findings and those of Minchev et al. (2012b) may arise in part due to the rapid heating apparent in their simulations due to a violent initial instability. At later times, the phenomenon driving the disc trends in their models (the same simulations were also discussed in Minchev et al. 2011, 2012a) is efficient angular momentum redistribution during bar growth (e.g. Hohl 1971) as opposed to more quiescent redistribution by spirals which dominates the evolution in our simulations (Roškar et al. 2012). Furthermore, we need to emphasize here that while it is clear that the stars in our simulations (and all others) heat vertically, the mechanism responsible for such heating in *N*-body models is not well understood (Sellwood 2013).

3.2 Disc flaring

An important consideration for any model that deals with the thickening of the stellar disc is the change in scaleheight with radius, i.e. disc flaring. While substructure bombardment is particularly efficient at thickening discs, it may also cause rather drastic flaring to occur (Kazantzidis et al. 2009). Such flaring is not typically seen in extragalactic observations of edge-on discs (de Grijs & van der Kruit 1996).

While, radial migration has also previously been associated with dramatic flaring (Minchev et al. 2012b), we show in Fig. 4 that this is not necessarily the case. In the top panel, we show the vertical

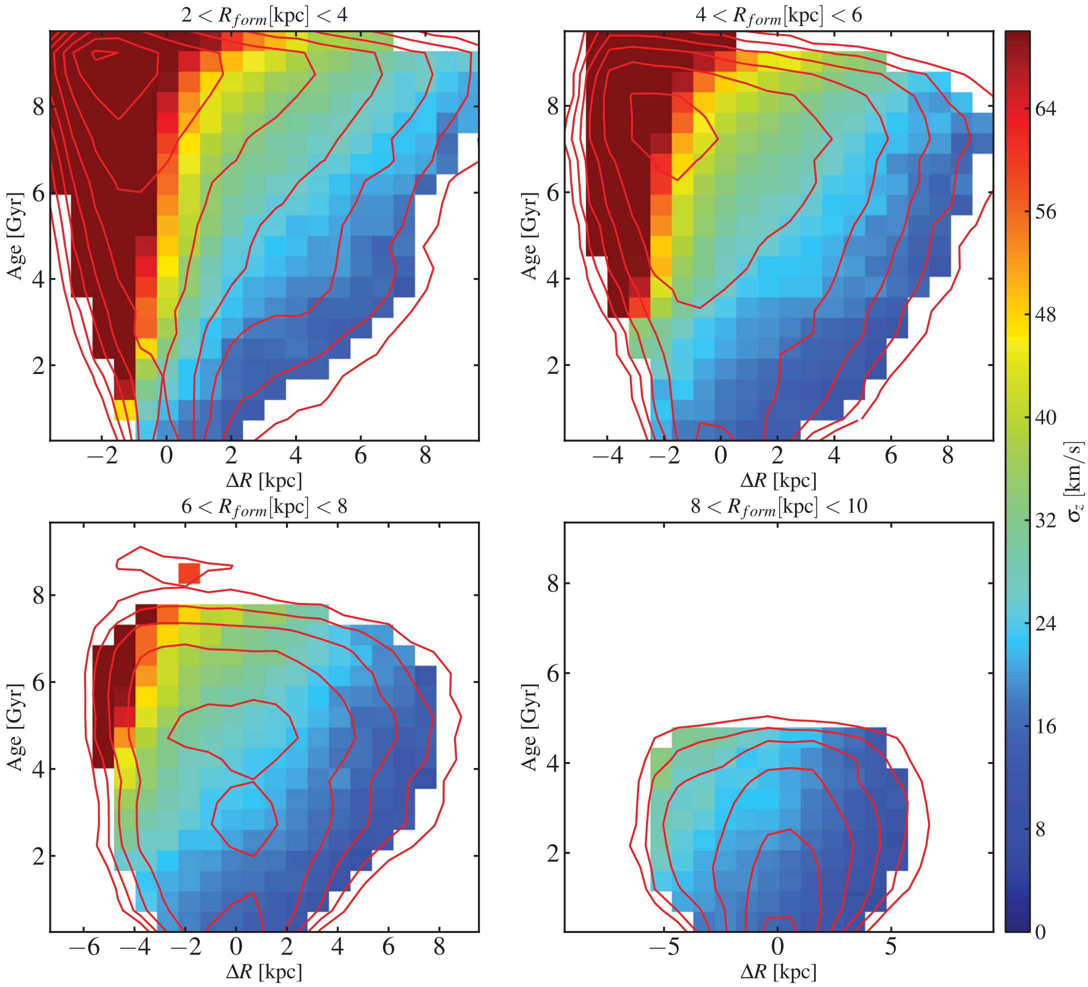

Figure 3. Same as Fig. 2 except here the colours correspond to σ_z, the vertical velocity dispersion at the end of the simulation. The old stars that the 'solar neighbourhood' (i.e. ∼8 kpc) from the interior of the disc have $\sigma_z \sim 40$–50 km s^{-1} in broad agreement with thick-disc values in the MW.

profiles of the disc at several different radii, all normalized to their values at 1 kpc from the plane to more easily compare the profiles furthest from the plane. The vertical distribution clearly becomes more extended in the outer parts of the disc, but the flaring is rather minor.

In the bottom panel, we quantify the flare by showing the values of maximum-likelihood estimates (MLE) for the scaleheight of the thicker component of a double sech^2 distribution as a function of radius. We use MLE instead of a least-squares fit to the binned vertical profiles because we found those fits to be unreliable, while MLE gives consistently good results. To find the MLE parameters, we maximize the log-likelihood function

$$\ell(h_1, h_2, f|z) = \sum_{i=0}^{N} \ln\left[\rho(z_i|h_1, h_2, f)\right] - \ln(\rho_0),$$

with

$$\rho(z)/\rho_0 = (1-f)\mathrm{sech}^2(z/2h_1) + f\,\mathrm{sech}^2(z/2h_2),$$

where ρ is the volume density, N is the total number of particles in the bin, z_i is the vertical position of the ith particle, h_1 and h_2 are the thin and thick disc scaleheights, f is the thick-disc fraction and ρ_0 is the mid-plane density. To estimate the parameters, we select particles found in 1 kpc radial bins centred on each point shown in Fig. 4 and up to 4 kpc from the mid-plane. Following Bovy et al. (2012a), the likelihood function ℓ is maximized using an MCMC sampler (Foreman-Mackey et al. 2012), yielding parameter estimates and uncertainties, represented by the error bars in Fig. 4.

The increase in scaleheight is only 50 per cent across approximately 10 kpc (corresponding to 4 disc scalelengths). The thick-disc fraction is ∼12–20 per cent in the main disc region (4–10 kpc) and rises to ∼40 per cent in the outer disc. Interior to 10 kpc, the scaleheight is essentially constant. Note that most studies fitting the vertical density of the MW disc (e.g. Gilmore & Reid 1983; Jurić et al. 2008; Bovy et al. 2012a) use an exponential fitting function and that the scaleheights between the two functional forms are not strictly comparable. We find that fitting our simulated particle distribution with an exponential model yields scaleheights larger by up to ∼30 per cent, making them only slightly smaller than similar thick-disc scaleheight measurements in the MW. However, we use the sech^2 because it gives a better fit to our model.

4 DISC THICKENING DUE TO CONSERVATION OF VERTICAL ACTION

Here, we consider the simplified 1D problem with a star oscillating above and below the plane of a thin disc. This is similar to the model

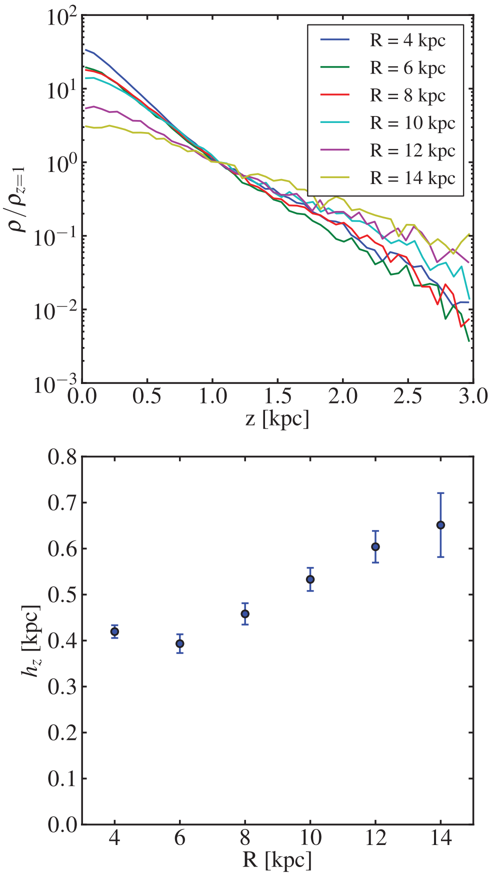

Figure 4. Top: vertical density profiles at several different radii, normalized to their values at $z = 1$. Bottom: scaleheights of the thicker sech^2 component as a function of radius. From the inner disc to the outermost regions we find a modest increase in scaleheight.

described by Minchev et al. (2012b), who use it to show that the vertical velocity dispersion is expected to decrease as stars migrate outwards if the vertical action is conserved. We show that this same model leads to the conclusion that the stellar vertical distribution *must* thicken. In this simple case, the vertical frequency is

$$\nu \propto \sqrt{\rho(R)}, \tag{1}$$

where $\rho(R) = \rho_0 \mathrm{e}^{-R/R_\mathrm{d}}$ is the mid-plane density of the disc at radius R, ρ_0 is the density at the disc centre and R_d is the disc scalelength. The vertical action can be approximated by $L_z = E_z/\nu$, where E_z is the vertical energy. For a population of stars, we assume that $E_z \sim \sigma_z^2$. If we assume that the vertical action L_z is conserved (Solway et al. 2012), then

$$\sigma_z^2 \propto \nu \propto \mathrm{e}^{-R/2R_\mathrm{d}}, \tag{2}$$

as shown previously by Minchev et al. (2012b).

The vertical density distribution of an isothermal population is given by (Spitzer 1942)

$$\rho(z) = \rho_{z=0}\,\mathrm{sech}^2(z/2h_z), \tag{3}$$

with the vertical scaleheight

$$h_z = \frac{\sigma_z^2}{2\pi G \Sigma(R)}. \tag{4}$$

Plugging equation (2) into equation (4), and assuming that the disc surface density $\Sigma \propto \mathrm{e}^{-R/R_\mathrm{d}}$, one finds that $h_z \propto \mathrm{e}^{R/2R_\mathrm{d}}$, i.e. the scaleheight *increases* with positive changes in radius. Combining the approximation for change in σ_z for a migrating population with equation 4 yields a simple expression relating the change in radius ΔR and R_d to the ratio of initial and final scaleheights, $h_{z,f}/h_{z,i}$:

$$\frac{h_{z,f}}{h_{z,i}} = \exp\left(\frac{\Delta R}{2R_\mathrm{d}}\right). \tag{5}$$

This simple derivation of equation (5) ignores the presence of other disc components; such an isothermal population is just one of many that make up the disc. In particular, the addition of mass (by gas accretion and star formation) in the mid-plane would adiabatically compress the migrated population, which has previously been shown to alleviate some of the thickening effects of infalling substructures (Moster et al. 2010; Villalobos, Kazantzidis & Helmi 2010). Furthermore, even in a disc not subject to outside perturbations, σ_z will also increase with time due to heating from disc structure thereby further boosting the scaleheight, an additional effect that we discuss below.

Equation (5) is an approximation to the more general expression derived by Schönrich & Binney (2012, equation [27]), which instead of the factor 2 on the right hand side includes a factor of $(2 + \alpha)$. The α changes depending on the magnitude of vertical oscillation. Note that the change induced in the z distribution due to adiabatic invariance occurs irrespective of the reason for the change in radius. The change may be due to radial oscillations about the guiding radius (Schönrich & Binney 2012) or due to the change in the guiding radius itself. Therefore, the increase in z_rms with increasing ΔR in Fig. 2 is due to a complex combination of radial heating, migration, and the evolution of the galactic potential.

In Fig. 5 we try to separate out the dependence of stellar population thickness on these processes. We focus on stars that are 5–6 Gyr old and formed within the range of specified radii. We then further separate these stars by their orbital circularity parameter, $j_z/j_c(E)$ measured at the present time, where j_z is the specific angular momentum and $j_c(E)$ is the maximum specific angular momentum for a star with a given energy. The solid lines show the variation in fractional change in thickness $z_\mathrm{rms}/z_{i,\mathrm{rms}}$ versus change in guiding radius since birth, ΔR_g, for different ranges in orbital circularity. The range in j_z/j_c is quite representative of this particular subset of stars, i.e. there are not many particles within the chosen constraints that are significantly kinematically hotter. We define R_g in terms of angular momentum, i.e. $R_\mathrm{g} = j_z/V_\mathrm{c}$, where $V_\mathrm{c} = 250\,\mathrm{km\,s^{-1}}$ is the approximate circular velocity. The age and R_form ranges are chosen to give a reasonable sampling of the main part of the disc.

Looking at Fig. 5, one can now identify the dependence of a stellar population's vertical distribution on heating and migration separately. For the kinematically coldest population, the thickness changes by a factor of 3 along the range of ΔR_g. On the other hand, it increases by a further factor of $\sim$2 from the coldest population to the hottest population, indicating that both processes actually contribute in similar amounts to the thickening.

The evolution of the thickness of this coeval stellar population can be described by the simple disc model discussed above under the assumptions of adiabatic invariance. We extend the simple formula in equation (5) to that derived by Schönrich & Binney (2012) and plot the result in Fig. 5 for different values of α shown with coloured dashed lines using a disc scalelength $R_\mathrm{d} = 2.5$ kpc. The least-squares best fit is shown with the red dashed line. Note that for a sech^2 distribution, z_rms is smaller than the scaleheight by approximately 10 per cent, so z_rms gives a reasonable approximation to z_0 that we can use in equation (5).

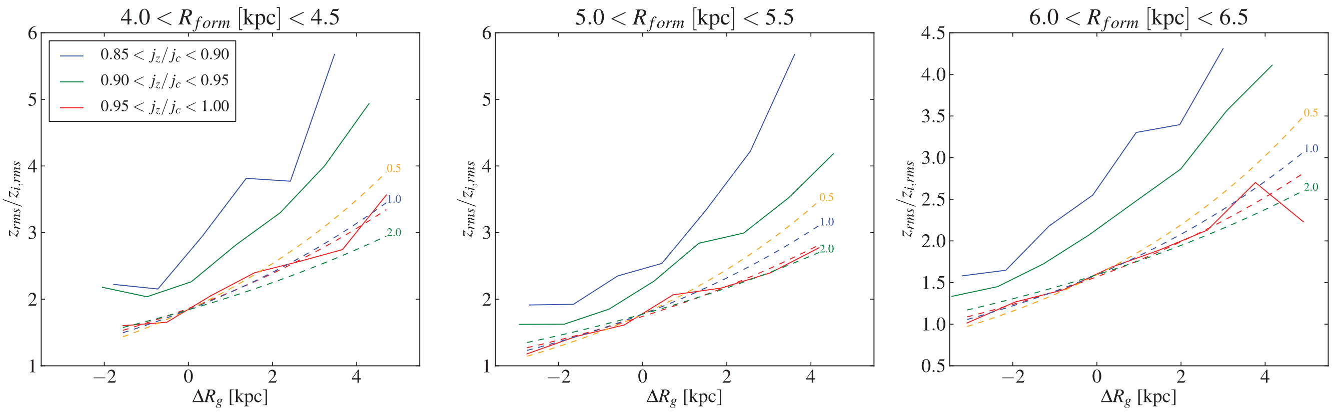

Figure 5. The ratio of $z_{\rm rms}$ (thickness now) to $z_{i,{\rm rms}}$ (thickness at formation) of 5–6 Gyr old stars that were formed within the indicated radial range, plotted against the change in guiding radius $\Delta R_{\rm g} = \Delta j_z/V_{\rm c}$ from their birth to the present, where Δj_z is the change in angular momentum and $V_{\rm c}$ is the circular velocity. The colours of the solid lines correspond to different ranges in the orbital circularity parameter $j_z/j_c(E)$. The overplotted red dashed lines are exponential fits, indicating that the changes in thickness satisfy the expectations from adiabatic invariance of the vertical action (see the text). The other coloured dashed lines show the relation from equation 27 of Schönrich & Binney (2012) for the kinematically coldest population, overplotted for different values of α and assuming a scalelength of 2.5 kpc.

The agreement between the simulation and the analytic prediction of the simple model, given the assumptions, is encouraging. Note that the likely value of α should be 1–2 since particles with the largest values of j_z/j_c have amplitudes of vertical oscillations $|z_{\rm max}| \ll R_{\rm d}$ (Schönrich & Binney 2012), which provides an excellent description of the simulation data. We have experimented with using different criteria for the $R_{\rm form}$ and age selection and found no appreciable differences, though the best-fitting parameters start to deviate from the expected values close to the centre or far in the outer disc where the surface density profile can no longer be described as exponential. The three different ranges of $R_{\rm form}$ shown in Fig. 5 show no appreciable differences. If we choose a different age range, the relations stay qualitatively the same.

Heating by spirals in our simulation results in increased random motion in-plane and out-of-plane. The increased thickness of the populations on slightly more eccentric orbits shown with the blue and green lines is therefore due to the combined effects of higher vertical random energy as well as amplified vertical oscillations due to adiabatic invariance at the orbital apocentres.

We have shown above that the thickness of a coeval population is an exponential function of its migration in the disc. It is interesting to pose a similar question for stars that share the same angular momentum *today* but that may have originated in different parts of the disc, i.e. are not to be considered a coeval population. As in Fig. 5, in the leftmost panel of Fig. 6, we show the ratio $z_{\rm rms}/z_{i,{\rm rms}}$ for stars with $5 < R_{\rm final}$[kpc] < 5.5 versus $\Delta R_{\rm g}$. We see similar exponential trends at all values of j_z/j_c, as before. Since the most circular (red line) population is the least affected by heating, we can once again use it as a proxy for the overall effect of radial migration on population thickness. Across the range of $\Delta R_{\rm g}$, we find that migration can lead to an thickness increase by a factor of 2.5 (we cannot eliminate all heating so thickness increases by 50 per cent simply by virtue of these stars spending 5 Gyr in the disc). Again, taking the range from blue to red as the effect of heating, we see that heating contributes another factor of $<$1.5. Based on this and Fig. 5, *we conclude that migration thickens stellar populations by about the same proportion as the internal heating processes, for parts of the disc outside the bulge.*

In our simulations, the gas cooling function does not include metal line or molecular cooling, limiting the temperature of the gas to $>10^4$ K. As a result, the gas velocity dispersion is largely set by the minimum temperature and consequently the gas scaleheight increases as a function of radius. Due to this gas flaring, the populations at each $\Delta R_{\rm g}$ in Fig. 6 are actually born with different thicknesses, as shown in the middle panel. The variation across the range of 5–6 Gyr old stars that end up at the specified angular momentum is a factor ∼2–3. Hence, while it is inevitable that individual populations change their vertical distribution as they migrate (Fig. 2), these changes are partially offset by the flaring of stars at birth (middle panel of Fig. 6). As a result, at a *fixed* final angular momentum (i.e. $R_{\rm g}$) there is little variation of thickness with $\Delta R_{\rm g}$.

However, the molecular and atomic gas components in the MW show little variation in thickness out to ∼10 kpc (Bronfman et al. 1988; Wouterloot et al. 1990; Narayan & Jog 2002). We may therefore expect that in the MW the flaring would not mask the thickening resulting from radial migration. Evidence for similarly constant scaleheights as a function of radius has also recently been found with resolved-star studies of edge-on nearby discs (de Jong & Streich, private communication).

We checked the results of this section against a higher resolution version of our simulation that uses four times as many particles (simulation R4 from Roškar et al. 2012) and found no significant differences, so we consider the dynamical results to be numerically robust. However, assessing the significance of the gas disc flaring and the subsequent radial dependence of the scaleheight of young stars is a much more difficult problem. It depends sensitively on the details of the gas physics on scales that are an order of magnitude smaller than what we (or any other similar state-of-the-art simulations at present) are able to adequately resolve.

5 CONSEQUENCES FOR LOCAL ABUNDANCE TRENDS

The consequence of the thickening described above on the observed properties of stellar populations is that their structural properties vary considerably with metallicity and abundance. This has been pointed out by Bovy et al. (2012a), who found that when they sliced the SEGUE G-dwarf sample into 'mono-abundance' bins, the structural parameters varied smoothly from short and thick at

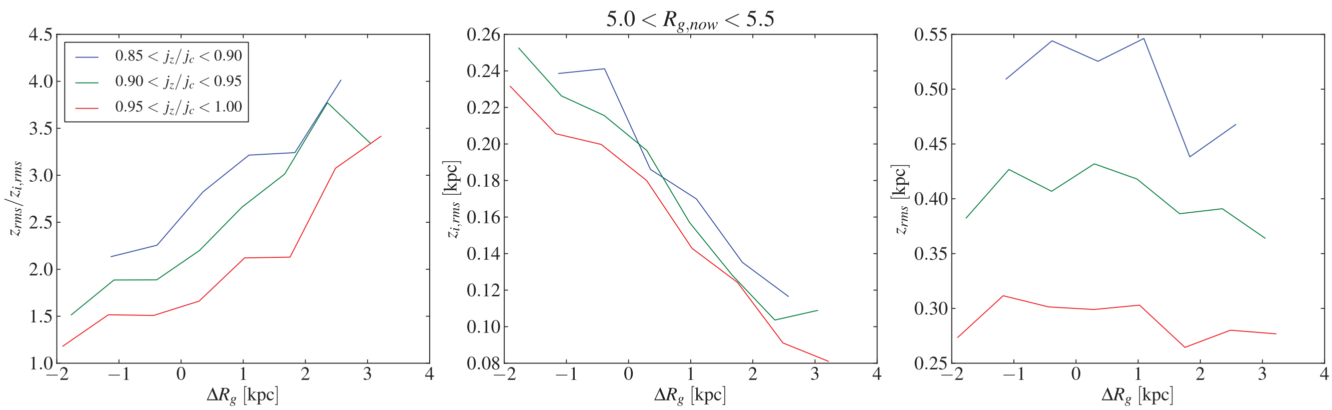

Figure 6. The change in population thickness as a function of $\Delta R_{\rm g}$, the change in guiding radius since birth, defined as $R_{\rm g} = j_z/V_{\rm c}$ (same as in Fig. 5) for particles with $R_{\rm g}$ in the indicated range. The left-hand panel shows the ratio of $z_{\rm rms}/z_{i,\rm rms}$, predicted by equation (5) to follow an exponential. The centre and right-hand panels show $z_{\rm rms}$ at time of formation and at the present time, respectively. Stars used in this figure are 5–6 Gyr old.

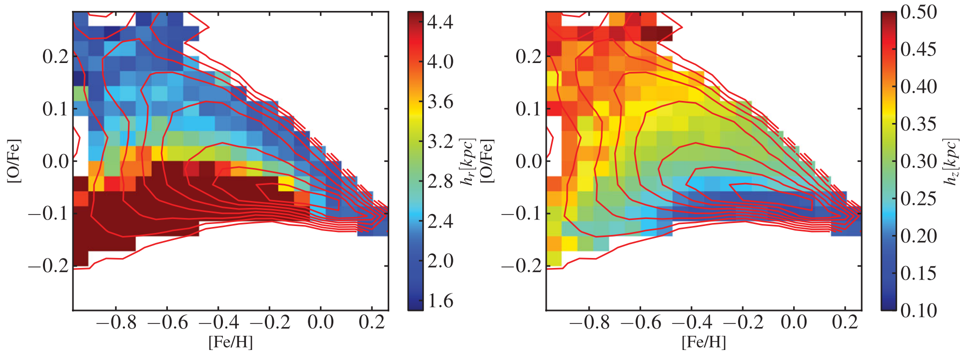

Figure 7. The distribution of stars in the [O/Fe] versus [Fe/H] plane, coloured by scalelength h_r on the left and scaleheight h_z on the right. The logarithmically spaced contours indicate mass density from 10 to 10^4 particles per bin. The stars lie between $4 < R\,[\rm kpc] < 9$ and $|z| < 3\,[\rm kpc]$.

the metal-poor α-rich end to long and thin at the metal-rich α-poor end. We show a similar dissection of our model disc stars in Fig. 7 and find a qualitatively similar result, i.e. the scaleheight decreases smoothly from upper left to lower right and the thinner populations tend to have longer radial scalelengths.

To obtain the structural parameters, we parametrize the stellar distribution as

$$\rho(R, z) \propto f(R) f(z),$$

where the radial and vertical density functions are

$$f(R) = \exp\left(-\frac{R}{h_R}\right),$$

and

$$f(z) = \mathrm{sech}^2\left(-\frac{z}{2h_z}\right),$$

respectively. We fit these functions to the simulated particle density distribution for the maximum likelihood values of h_r and h_z using a procedure similar to the one described in Section 3.2. We use stars within a range of $4 < R < 9$ kpc and $|z| < 3$ kpc to obtain the fits. By requiring at least 100 particles per each plotted cell, we obtain 1σ uncertainties that are everywhere less than 10 per cent. As in Section 3.2, we experimented also with an exponential function for the vertical distribution, and found that we can obtain smaller errors using the sech^2 form. If we fit the vertical distributions of stellar populations in our model with a single exponential, we obtain scaleheights that are somewhat higher than those shown in Fig. 7, similar to what we found in Section 3.2.

The locus of 'saturated' scalelengths in Fig. 7 below [O/Fe] ~ -0.05 are all young stars. Their scalelengths are long because they are forming out of the cold gas which is also very extended. A similar saturation is seen in Bovy et al. (2012a). In our model, the sub-solar part of this locus originates in the outer disc, and Bovy et al. (2012a) similarly find that the mean radii for these stars are beyond the solar neighbourhood. These are young, 0–2 Gyr old stars that formed in the outer disc and scattered into the solar neighbourhood (see also Haywood 2008). They did not migrate via the corotation mechanism, because their tangential velocities exceed the circular velocity by $\sim$20 km s^{-1}. Therefore, they *heated* and although one would naively expect their negative ΔR to lead to a thinner scaleheight, their scaleheights are slightly higher than the local young population by virtue of this heating. We can identify this population in the lower-right panel of Fig. 2 as the 2-Gyr-old stars that migrate inwards by $\sim$1–2 kpc and find that their scaleheight $h_z \sim 0.3$ kpc.

An interesting discrepancy with the Bovy et al. (2012a) distribution is seen at the extreme metal-rich, α-poor corner ([O/Fe] $<$ -0.1 and [Fe/H] > 0.1). We find that those stars have a short scalelength much like the older, more α-rich population. In Bovy et al. (2012a), no such population exists. In our model, these stars are dominated by intermediate-age populations, mostly around $\sim$3 Gyr and 5–6 Gyr old. The majority (75 per cent) of these stars came from inside 3 kpc, i.e. from the same part of the disc as some of the oldest stars in the solar neighbourhood. These are the prototypical 'extreme' migrators that were able to migrate far because they never heated considerably. Hence, their overall velocity dispersion is ∼5–10 km s^{-1} lower than other stars of similar age and because of the lower heating they remain thinner than the average population of the same age. Note that to increase particle numbers for more reliable fitting, we extended the selection in Fig. 7 to include stars to 4 kpc; this population still exists when we restrict the sample to 7–9 kpc. We speculate that should such stars exist in the MW, they may be absent from the SEGUE sample due to their thin distribution and low numbers. However, should they be observed in future surveys, they would be an indication of significant radial mixing in the MW disc. An enticing connection to such a migrated population may be the newly discovered metal-rich, high-α stars (Adibekyan et al. 2011, 2013; Gazzano et al. 2013) whose chemical properties suggest that they formed in the Galactic interior, but they are kinematically akin to local thin-disc stars.

If the local abundance and metallicity distributions are affected by stars migrating from the inner disc, one would expect to find corresponding populations in the present-day bulge. The thickest parts of the local thickened disc have low [Fe/H] and high α abundance, which may correspond qualitatively to a bulge population 'C' identified by Ness et al. (2013). The timing of the migration is difficult to predict, though in general stars can migrate more easily before they heat considerably. On the other hand, the bulge region seems to host stars of all abundances, metallicities and ages (Bensby et al. 2013), limiting the constraints one could impose on the models. In contrast to the local disc, however, the inner Galaxy does show a strong age–metallicity relation. Therefore, if the thickened population at the Sun's position is old and metal-poor, it could be broadly consistent with being migrated from the inner disc/bulge. Determining whether migration should be considered critical for the evolution of the MW will be determined in part by upcoming chemical tagging surveys such as HERMES (Bland-Hawthorn, Krumholz & Freeman 2010).

6 CONCLUSIONS

We have demonstrated that an outward migrating population of stars in a galactic disc *always* thickens. At the same time, the vertical velocity dispersion of such a population decreases, but we find that despite this decrease the old stellar populations arriving from the interior at the solar neighbourhood match the kinematics of the MW thick disc. Furthermore, we find that radial migration and internal heating thicken coeval stellar populations by comparable amounts. The thickening due to radial migration alone is well approximated by a simple analytic treatment that assumes a conservation of vertical action. Importantly, we find that while radial migration does cause an increase in scaleheight with radius, the flaring that results is minor. If radial migration can contribute to a thickened component of a galactic disc, then it has important consequences in the broader context of disc galaxy formation since most other mechanisms proposed to form a thick-disc component involve the cosmological environment.

Our simulation recovers the qualitative structural stellar population trends observed in the MW SEGUE data. In particular, we find a similar dependence of scaleheight and scalelength on oxygen abundance and metallicity to those found in Bovy et al. (2012a). However, our model produces a thick disc that is somewhat too thin compared to the MW. This is to be expected, since the disc we considered in this work is built up entirely from internal mechanisms in the absence of a cosmological environment. Additional perturbations in a more realistic setting would thicken the disc further.

Finally, we identify an interesting sub-population in the low-[O/Fe] high-[Fe/H] corner of the abundance plane (Fig. 7), which appears to be absent from the Bovy et al. (2012a) distribution, perhaps due to the lack of low-latitude coverage in the SEGUE data. These stars are the prototypical extreme-migrators having migrated from ∼3 kpc in a few Gyr.

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