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Article

Survival Signature-based Reliability Approach for Complex Systems Susceptible to Common Cause Failures

Feng, Geng

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Survival Signature Based Reliability Approach for Complex Systems Susceptible to Common Cause Failures

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The importance of reliability to complex systems cannot be disputed as they are the backbones of our society. In practice, the common cause failures may have severe reverse function on complex systems' overall stability. Survival Signature opens a new way to perform reliability analysis on systems with multiple component types. This paper takes a research on survival signature based reliability analysis on complex systems susceptible to Common Cause Failures. To be specific, it proposes the standard α -factor model and general α -factor model to combine with the survival signature. In practical applications, the α -factor estimator of the system might not be defined completely due to limited data, or knowledge which requires to take imprecision into account. Some numerical cases are presented to show the applicability of the methods for complex systems. In addition, this paper may attract people's attention on the conception of Design for Reliability.

Keywords: Common Cause Failures; Survival Signature; α -Factor Model; Reliability Analysis; Complex Systems.

1. Introduction

A common cause failure (CCF) is an event that causes multiple components fail simultaneously, which exists widely in complex systems. Modelling CCFs is essential in complex systems reliability analysis as they can have a large effect on the systems' overall functionality, especially on the quality and productivity of products and processes.¹ There is always an assumption that the components failures are independent, however, CCFs make the hypothesis is not set up. What is more, CCFs have been shown by many studies to decrease the reliability and availability of complex systems. Therefore, common cause failures are extremely important in reliability assessment and must be given adequate treatment to minimize gross overestimation of performances.²

A number of parametric models have been developed for common cause failures over the last decades. Rasmuson and Kelly reviewed the basic concepts of modelling CCFs in reliability and risk studies.³ One of the most commonly used

single parameter models defined by Fleming⁴ is called the β -factor model, which is the first parameter model applied to common cause failures in risk and reliability analysis. Then he generalized the β -factor model to multiple Greek letter model in 1986.⁵ The α -factor model which is proposed by Mosleh et al.^{6,7} develops CCFs from a set of failure ratios and the total component failure rate. The binomial failure rate model discussed by Atwood⁸ estimates the failure frequency of two or more components in a redundant system as the product of the CCF shock arrival rate and the conditional failure probability of components given the shock has occurred. **A system's components each contribute differently to the system, which are defined as weights.⁹ Systems with weighted components are useful to model various capacity-based engineering systems such as oil transportation system, power generation system, and production system.¹⁰ Different from weighted reliability components measure, which mainly focus on the influence of different weighted components on the reliability of system, α -factor model is mainly used for estimating the probability of how many components fail due to a common cause failure event.**

Recently, based on the α -factor model, Kelly and Atwood¹¹ presented a method for developing Dirichlet prior distributions that have specified marginal means, but which are otherwise minimally informative. An explicit method and an implicit method is proposed in¹² to analyse the reliability of systems subject to internal or external probabilistic common cause failures. **A general applicable and efficient Monte Carlo simulation approach is used for reliability evaluation of systems susceptible to induced failures in.¹³** Levitin¹⁴ incorporated the common cause failures into non-repairable multistate series-parallel system analysis by using the universal generating function method. A robust Bayesian approach to modelling epistemic uncertainty in the imprecise Dirichlet model has been discussed by Troffaes et al.¹⁵ Coolen and Coolen-Maturi¹⁶ presents non-parametric predictive inference for system reliability following common cause failures of components. In that paper, attention is restricted to system with exchangeable or a single type of components. Therefore, an extension of this work is needed. To be specific, it is necessary to perform reliability analysis on complex systems by considering CCFs among components belonging to different types. Survival signature provides a good way solve this problem.

Survival signature was first proposed by Coolen and Coolen-Maturi¹⁷ in 2012. It is a powerful methodology that can not only hold the merits of the former system signature,¹⁸ but can be used in complex system with components belong to multiple types. In essence, it does not have the assumption that components of different types are exchangeable, which overcomes the long-standing limitation of the system signature. This is useful when a system which may have components with failure times that follow different probability distributions.^{19,20} Therefore, survival signature is a promising method for application to complex systems and networks.

Based on the former work, Aslett developed a Reliability Theory package which

was used to calculate the survival signature²¹ and analysed system reliability within the Bayesian framework of statistics.²² A non-parametric predictive inference for system reliability using the survival signature was proposed by Coolen et al.²³ Feng et al. deals with the imprecision within the system by analytical and numerical ways respectively,²⁴ what is more, probability bounds analysis is used to perform sensitivity analysis on complex systems with epistemi uncertainty in the paper.²⁵ An imprecise Bayesian non-parametric approach by using sets of priors to system reliability with multiple types of components is developed by Walter et al.²⁶ Patelli et al.²⁷ proposed efficient simulation approaches which are based on survival signature for reliability analysis on large system. An efficient algorithm for the exact computation of the survival signature of large systems was put forward by Reed.²⁸ Eryilmaz et al.²⁹ developed the survival signature-based marginal and joint reliability importance measures to optimise system design, as well as consider mean residual life of coherent systems with multiple types of dependent components in the article.³⁰

A survival signature based reliability analysis on complex systems with common cause failures has been proposed in this paper. The α -factor model distinct between the total failure rate of a component and the common cause failures modelled by α -factor parameters, which can be got through experts' judgement of the system or the past data on the system. These advantages of the α -factor model make it possible to combine with the survival signature to assess the complex system reliability. The standard α -factor model is proposed first in the paper to perform system reliability analysis after CCFs, however, it has two assumptions that there must be at least one component fails of each type and the components failures of each type are independent. In order to remove these assumptions, another general α -factor model has been introduced. It takes all the possible combinations of the failed numbers of past events. The novel standard and general α -factor models can be expressed by the equations which connect with the survival signature. What is more, the imprecision within the models are also considered. The applicability of the proposed approaches are demonstrated by solving the numerical cases, by investigating the results, the concept of design for reliability should be drawn more attention.

This paper is organized as follows. Section 2 gives a brief conceptions about the α -factor model and survival signature. The system reliability analysis after common cause failures has been presented in Section 3. In this Section, the standard α -factor model and the general α -factor model have been studied respectively. The applicability and performance of the proposed approaches is presented in Section 4. Finally Section 5 closes the paper with conclusions and discussion.

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2. α -Factor Model and Survival Signature

2.1. α -factor model

The α -factor model is particularly useful in the practical engineering world as the alpha factor parameters can be got through experts' judgement of the system or past data on the system. The parameters α_k of the model are the fractions of the total probability of failure in the system that involves the failure of k components due to a common cause.

The probability of a common cause basic event involving failure of k components in a system of m components³¹ can be calculated by Equation 1.

$$Q_k = \frac{k}{\binom{m-1}{k-1}} \frac{\alpha_k}{\alpha_t} Q_t \quad (1)$$

where, $k = 1, 2, \dots, m$ and $\alpha_t = \sum_{k=1}^m k\alpha_k$. Q_t is the total probability of failure accounting both for common cause failures and independent failures. The alpha parameter estimator can be expressed as:

$$\alpha_k = \frac{n_k}{\sum_{i=1}^m n_i} \quad (2)$$

where, n_k is the number of events with k failed components.

The alpha parameter estimator represents the probability that exactly k of the m components fail, given that at least one failure occurs. It can be seen from Equation 2 that the sum of the α_k will be 1. The advantage of the α -factor model is its distinction between the total failure rate of a component Q_t , for which we generally have a lot of information, and common cause failures modelled by α_k , for which we generally have very little information.¹⁵

2.2. Survival signature

Suppose there is a system with m components which belong to $K \geq 2$ component types, with m_k components of type $k \in \{1, 2, \dots, K\}$ and $\sum_{k=1}^K m_k = m$. Assume that the random failure times of components of the system type are exchangeable, while full independence is assumed for components belong to different types (*iid*), Coolen¹⁷ proposed the survival signature which can be denoted by $\Phi(l_1, l_2, \dots, l_K)$, with $l_k = 0, 1, \dots, m_k$ for $k = 1, 2, \dots, K$. It defines the probability that the system functions given that l_k of its m_k components of type k work, for each $k \in \{1, 2, \dots, K\}$. There are $\binom{m_k}{l_k}$ state vectors $\underline{x}^k$ with $\sum_{i=1}^{m_k} x_i^k = l_k$ ($k = 1, 2, \dots, K$), where $\underline{x}^k = (x_1^k, x_2^k, \dots, x_{m_k}^k)$. Let $S_{l_1, l_2, \dots, l_K}$ denote the set of all state vectors for the whole system, and it can be known that all the state vectors $\underline{x}^k \in S_{l_k}^k$ are equally likely to occur. Therefore, the survival signature can be expressed as:

$$\Phi(l_1, \dots, l_K) = \left[\prod_{k=1}^K \binom{m_k}{l_k} \right]^{-1} \times \sum_{\underline{x} \in S_{l_1, \dots, l_K}} \phi(\underline{x}) \quad (3)$$

Let $C_k(t) \in \{0, 1, \dots, m_k\}$ denote the number of k components working at time t . Assume that the components of type k have a known cumulative distribution function (CDF) $F_k(t)$ and the components failure times of different type are assumed independent, then:

$$P\left(\bigcap_{k=1}^K \{C_k(t) = l_k\}\right) = \prod_{k=1}^K P(C_k(t) = l_k) = \prod_{k=1}^K \binom{m_k}{l_k} [F_k(t)]^{m_k - l_k} [1 - F_k(t)]^{l_k} \quad (4)$$

Hence, the survival function of the system with K types of components becomes:

$$P(T_s > t) = \sum_{l_1=0}^{m_1} \dots \sum_{l_K=0}^{m_K} \Phi(l_1, \dots, l_K) P\left(\bigcap_{k=1}^K \{C_k(t) = l_k\}\right) \quad (5)$$

Equation 5 shows that the structure of the system is separated from the its components failure times, which is the typical advantage of the survival signature. The survival signature is a summary of structure functions and only needs to be calculated once for the same system. As a result, it is an efficient method to perform system reliability analysis on complex systems with multiple component types.

3. System Reliability after Common Cause Failures

In this section, the survival signature is introduced to analyse complex system with common cause failures. Based on the results of,¹⁶ the α -factor model can be applied to calculate system reliability in the presence of common cause failures.

3.1. Standard α -factor model

Let assume that there is a system with m_k components belong to type $k \in \{1, 2, \dots, K\}$. When a failure event occur, $P(f_1, f_2, \dots, f_K)$ denotes the probability that how many failures occur of each component type. The survival signature $\Phi(l_1, l_2, \dots, l_K)$ is the probability that the system functions if l_k components of type k are working, which can be expressed as $\Phi(l_1, l_2, \dots, l_K) = P(\text{system functions} \mid l_k \text{ components of type } k \text{ work})$, where $k \in \{1, 2, \dots, K\}$.

Let $P(T_s > t \mid CCF)$ express the probability that the system still functions after the next common cause failure event, therefore, it can be calculated by

$$P(T_s > t \mid CCF) = \sum_{l_1=0}^{m_1} \dots \sum_{l_K=0}^{m_K} \Phi(l_1, \dots, l_K) P(m_1 - l_1, \dots, m_K - l_K) \quad (6)$$

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where $f_i = m_i - l_i$.

It can be seen from Equation 6 that unlike the $P(T_s > t)$ calculated by Equation 5, the survival function of the system after the CCFs is independent of t . What is more, the typical merit of survival signature can also hold. To be specific, the survival signature, which encompasses information of the system structures, and the probability distribution, which relies on the common cause failures of the components belong to different types, is separated in the equation.

The α -factor model with estimate of the alpha factor parameters are mainly used in this section. And they are given by Equation 2 for a common cause group of m components.

$P(f_1, f_2, \dots, f_K)$ is constructed by using the past data available on the system, combined with the α -factor model for CCFs. For this standard α -factor model, there are assumptions that $P(0, 0, \dots, 0) = P(0, f_2, \dots, f_K) = P(f_1, 0, \dots, f_K) = \dots = P(f_1, f_2, \dots, 0) = 0$, as there must be at least one component fails of each type. What is more, there is another assumption that the components failures of each type are independent, which implies that $P(f_1, f_2, \dots, f_K) = P(f_1) * P(f_2) * \dots * P(f_K)$.

Recall that $\alpha_{n_k}^k$ gives the probability that exactly n_k components fail, given that they belong to type k . For example, it is assumed that there is 1 component of type 1 failure, it follows that $P(f_1 = 1) = \alpha_1^1$. Thus, the alpha parameters provide all of the information required to specify the distribution. For instance, $P(1, 1, \dots, 1) = \alpha_1^1 \alpha_1^2 \dots \alpha_1^K$.

The next step is to calculate the survival signature, which will then combine with the $P(f_1, f_2, \dots, f_K)$ to assess the survival function of the system. It is not necessary to computer all the survival signatures as it can identify which values are required by Equation 6.

3.2. General α -factor model

The standard α -factor model may lead to an unsatisfactory low probability level of the system reliability, which due to the assumption that the first failure event will affect components of all types. **Therefore, it is essential to find a general α -factor model that can avoid the assumption that the failure event has necessarily affected all common cause groups of components.**

Let assume there are K common cause component groups, with group $k \in \{1, 2, \dots, K\}$ includes m_k components. The past data $n_{j_1, j_2, \dots, j_K}$ denotes the number of past events with exactly j_1 failed components from group 1, exactly j_2 failure components from group 2 and the like.

The α -factor parameter $\alpha_{j_1, j_2, \dots, j_K}$ provides the probability that exactly j_k components of group k fail due to a common cause failure event, with $k \in \{1, 2, \dots, K\}$. Given that a common cause failure event has affected the overall system, however, which exact common cause groups are affected are not known. Therefore, the $\alpha_{j_1, j_2, \dots, j_K}$ can be estimated by Equation 7.

$$\alpha_{j_1, \dots, j_K} = \frac{n_{j_1, \dots, j_K}}{(\sum_{j_1=0}^{m_1} \dots \sum_{j_K=0}^{m_K} n_{j_1, \dots, j_K}) - n_{0, \dots, 0}} \quad (7)$$

The $\sum_{j_1=0}^{m_1} \dots \sum_{j_K=0}^{m_K} n_{j_1, \dots, j_K}$ in denominator represents all the possible outcomes, but $n_{0, \dots, 0}$ has to be subtracted as there is an assumption that at least one component of type k in the system is affected by the common cause failure event. However, this assumption is only valid if $n_{0, \dots, 0}$ is included in the summation. In practice, there may be no data for the number of no components have failed, therefore, for this circumstance, $n_{0, \dots, 0}$ is arbitrarily setted as 0.

The probability of the system works after a common cause failure event can also be expressed by Equation 6. The survival signature $\Phi(l_1, \dots, l_K)$ will not change for the same system, as it only depends on the structures of the system. However, the joint probability distribution $P(f_1, f_2, \dots, f_K)$ is obtained without assuming independence among components, as it is simply given by the alpha factor parameters. To be specific, that means $P(f_1, f_2, \dots, f_K) = \alpha_{j_1, j_2, \dots, j_K}$.

3.3. Imprecise system reliability after common cause failures

In the application engineering world, if the system has not been in operation in the past (e.g., is a new system), or it does not usually encounter common cause failure events. All of which means there may not be enough data or perfect inspections to estimate the values of the accurate α -factor parameters.³² In addition, experts can provide a estimation of the number n_k of common cause components failures, but often only an interval is predicted.

Therefore, the α -factor parameters might have imprecise values with $[\underline{\alpha}_{m_k}^k, \bar{\alpha}_{m_k}^k]$. Since $P(f_1, f_2, \dots, f_K)$ is estimated based on the α -factor model, the imprecision will propagate to $P(m_1 - l_1, \dots, m_K - l_K)$. The survival signature remains unaffected by the imprecision since it is only influenced by uncertainty and imprecision in the system structure. Hence, the bounds of the survival probability are calculated as

$$\underline{P}(SCCF) = \sum_{l_1=0}^{m_1} \dots \sum_{l_K=0}^{m_K} \Phi(l_1, \dots, l_K) \underline{P}(m_1 - l_1, \dots, m_K - l_K) \quad (8)$$

$$\bar{P}(SCCF) = \sum_{l_1=0}^{m_1} \dots \sum_{l_K=0}^{m_K} \Phi(l_1, \dots, l_K) \bar{P}(m_1 - l_1, \dots, m_K - l_K) \quad (9)$$

4. Numerical Examples

Figure 1 shows a complex system with thirteen components which belong to four types.

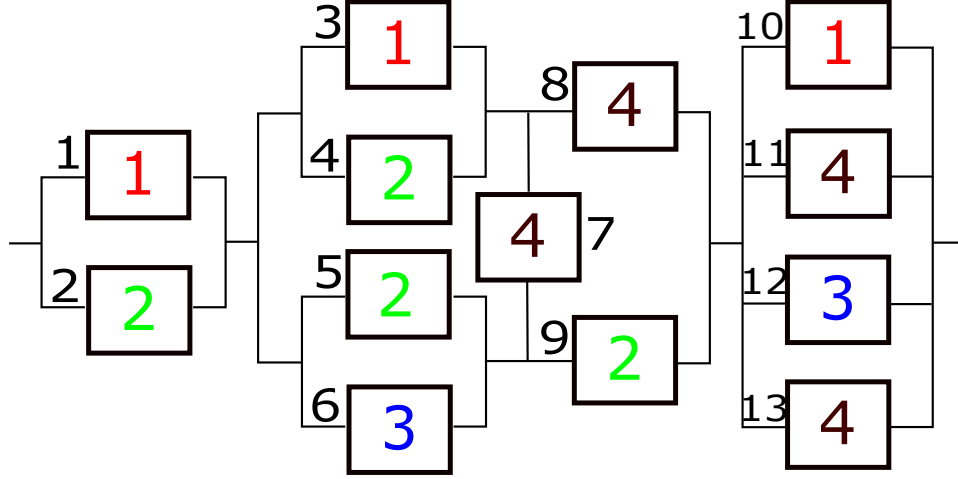


Fig. 1. Complex system with thirteen components which belong to four types. The number inside the component box represents the type, while the number outside the box expresses the component index.

4.1. Case 1: standard α -factor model

Firstly, let determine the values of the survival function by using the standard α -factor model with past data on the complex system. Suppose that the common cause failure groups are the same as component types, this is logical in engineering world as the components of the same type have similar characteristics, therefore, they are more likely to be influenced by the same common cause event.

There have $n_1 = 1$, $n_2 = 2$, $n_3 = 1$ for component type 1. For example, $n_1 = 1$ means that there has been 1 previous occurrences of failure with just one component of type 1. So the α -factor parameter estimator in this case is

$$\alpha_1^1 = \frac{n_1}{n_1 + n_2 + n_3} = \frac{1}{4} \quad (10)$$

It can also get the results that $\alpha_2^1 = \frac{1}{2}$ and $\alpha_3^1 = \frac{1}{4}$ in the same way. Just as the said before in this paper, it can be seen the summation of the α -factor parameters is equal to 1.

For type 2, $n_1 = 2$, $n_2 = 1$, $n_3 = 1$ and $n_4 = 2$, which gives $\alpha_1^2 = \frac{1}{3}$, $\alpha_2^2 = \frac{1}{6}$, $\alpha_3^2 = \frac{1}{6}$ and $\alpha_4^2 = \frac{1}{3}$ respectively.

There are $n_1 = 2$ and $n_2 = 1$ for type 3. Thus, $\alpha_1^3 = \frac{2}{3}$ and $\alpha_2^3 = \frac{1}{3}$.

Similarly for component type 4, the data of $n_1 = 3$, $n_2 = 3$, $n_3 = 1$ and $n_4 = 1$ lead to $\alpha_1^4 = \frac{3}{8}$, $\alpha_2^4 = \frac{3}{8}$, $\alpha_3^4 = \frac{1}{8}$ and $\alpha_4^4 = \frac{1}{8}$.

There has an assumption that a common cause failure event occur will affect at least one component of each component type for the standard α -factor model, therefore, it can be known for this system that $P(0, 0, 0, 0) = P(0, b, c, d) = P(a, 0, c, d) = P(a, b, 0, d) = P(a, b, c, 0) = 0$, for $a = 1, 2, 3$, $b = 1, 2, 3, 4$, $c = 1, 2$ and $d = 1, 2, 3, 4$.

There is another assumption that the failure components of different types are independent, which implies that $P(f_1, f_2, f_3, f_4) = P(f_1)P(f_2)P(f_3)P(f_4)$. According to the conception, $P(f_1, f_2, f_3, f_4)$ is calculated by the available past data on the system, and is combined with the α -factor model for common cause failures.

Recall that α_1^1 represents the probability that exactly one component of the three components of type 1 failure. Since it has been assumed that one component of type 1 fails, it follows that $P(f_1) = \alpha_1^1$. So the alpha parameters provide all the information that required to specify the distribution. For instance, $P(1, 1, 1, 1) = \alpha_1^1 \alpha_1^2 \alpha_1^3 \alpha_1^4 = \frac{1}{4} * \frac{1}{3} * \frac{2}{3} * \frac{3}{8} = \frac{1}{48}$.

Then, it is necessary to calculate the survival signature Φ , which is used to combine with the $P(f_1, f_2, f_3, f_4)$ to assess the survival probability of the system. For $P(1, 1, 1, 1)$, its corresponding survival signature is $\Phi(2, 3, 2, 3)$, which means that probability of the system works given that exact 2 components of type one, 3 components of type two, 2 components of type three and 3 components of type four are working. There are altogether 48 possible state vectors, of which 41 combinations allow the system to function. Therefore, $\Phi(2, 3, 2, 3) = \frac{41}{48}$.

All the values of $P(f_1, f_2, f_3, f_4)$ and their corresponding survival signature $\Phi(m_1 - f_1, m_2 - f_2, m_3 - f_3, m_4 - f_4)$ can be calculated. Based on the values and Equation 6, the probability that the complex system works after the next common cause failure event $P(T_s > t | CCF)$ is $\frac{50}{139}$.

4.2. Case 2: general α -factor model 1

Let continue use the complex system in Figure 1, however, without assuming that the common cause event will affect all the common cause failure groups. Again, the component types form the common cause failure groups as before. Therefore, it can be any number combinations of the components from type 1 to type 4. Note again that $P(0, 0, 0, 0) = 0$ as at least one component will be affected by the CCFs.

The past data on the system can be seen in Table 1, which has more data available to fit the requirements for implementing the general α -factor model.

Table 1: Past data $n_{j_1 j_2 j_3 j_4}$ on the system in Figure 1

$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$
$n_{0001} = 1$	$n_{0002} = 1$	$n_{0003} = 1$	$n_{0004} = 1$	$n_{0010} = 1$	$n_{0011} = 1$
$n_{0012} = 1$	$n_{0013} = 1$	$n_{0014} = 1$	$n_{0020} = 1$	$n_{0021} = 1$	$n_{0022} = 1$
$n_{0023} = 1$	$n_{0024} = 1$	$n_{0100} = 1$	$n_{0101} = 1$	$n_{0102} = 3$	$n_{0103} = 1$
$n_{0104} = 1$	$n_{0110} = 1$	$n_{0111} = 1$	$n_{0112} = 1$	$n_{0113} = 1$	$n_{0114} = 1$
$n_{0120} = 1$	$n_{0121} = 1$	$n_{0122} = 1$	$n_{0123} = 1$	$n_{0124} = 2$	$n_{0200} = 1$
$n_{0201} = 1$	$n_{0202} = 1$	$n_{0203} = 1$	$n_{0204} = 1$	$n_{0210} = 1$	$n_{0211} = 2$
$n_{0212} = 1$	$n_{0213} = 1$	$n_{0214} = 1$	$n_{0220} = 1$	$n_{0221} = 1$	$n_{0222} = 1$
$n_{0223} = 1$	$n_{0224} = 1$	$n_{0300} = 1$	$n_{0301} = 1$	$n_{0302} = 1$	$n_{0303} = 1$
$n_{0304} = 1$	$n_{0310} = 2$	$n_{0311} = 2$	$n_{0312} = 1$	$n_{0313} = 1$	$n_{0314} = 1$

Continued on next page

Table 1 – *Continued from previous page*

$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$
$n_{0320} = 1$	$n_{0321} = 1$	$n_{0322} = 1$	$n_{0323} = 1$	$n_{0324} = 1$	$n_{0400} = 1$
$n_{0401} = 1$	$n_{0402} = 1$	$n_{0403} = 1$	$n_{0404} = 1$	$n_{0410} = 1$	$n_{0411} = 2$
$n_{0412} = 1$	$n_{0413} = 1$	$n_{0414} = 1$	$n_{0420} = 1$	$n_{0421} = 1$	$n_{0422} = 1$
$n_{0423} = 1$	$n_{0424} = 1$	$n_{1000} = 1$	$n_{1001} = 1$	$n_{1002} = 1$	$n_{1003} = 1$
$n_{1004} = 1$	$n_{1010} = 1$	$n_{1011} = 1$	$n_{1012} = 2$	$n_{1013} = 1$	$n_{1014} = 1$
$n_{1020} = 1$	$n_{1021} = 1$	$n_{1022} = 1$	$n_{1023} = 1$	$n_{1024} = 1$	$n_{1100} = 1$
$n_{1101} = 1$	$n_{1102} = 1$	$n_{1103} = 2$	$n_{1104} = 1$	$n_{1110} = 2$	$n_{1111} = 1$
$n_{1112} = 1$	$n_{1113} = 1$	$n_{1114} = 1$	$n_{1120} = 1$	$n_{1121} = 1$	$n_{1122} = 1$
$n_{1123} = 1$	$n_{1124} = 1$	$n_{1200} = 1$	$n_{1201} = 2$	$n_{1202} = 1$	$n_{1203} = 1$
$n_{1204} = 1$	$n_{1210} = 1$	$n_{1211} = 1$	$n_{1212} = 1$	$n_{1213} = 1$	$n_{1214} = 1$
$n_{1220} = 1$	$n_{1221} = 3$	$n_{1222} = 2$	$n_{1223} = 1$	$n_{1224} = 1$	$n_{1300} = 1$
$n_{1301} = 1$	$n_{1302} = 1$	$n_{1303} = 1$	$n_{1304} = 1$	$n_{1310} = 1$	$n_{1311} = 3$
$n_{1312} = 1$	$n_{1313} = 1$	$n_{1314} = 1$	$n_{1320} = 1$	$n_{1321} = 1$	$n_{1322} = 1$
$n_{1323} = 1$	$n_{1324} = 1$	$n_{1400} = 1$	$n_{1401} = 1$	$n_{1402} = 1$	$n_{1403} = 1$
$n_{1404} = 2$	$n_{1410} = 4$	$n_{1411} = 1$	$n_{1412} = 1$	$n_{1413} = 1$	$n_{1414} = 1$
$n_{1420} = 1$	$n_{1421} = 1$	$n_{1422} = 1$	$n_{1423} = 1$	$n_{1424} = 1$	$n_{2000} = 1$
$n_{2001} = 1$	$n_{2002} = 1$	$n_{2003} = 1$	$n_{2004} = 1$	$n_{2010} = 3$	$n_{2011} = 1$
$n_{2012} = 1$	$n_{2013} = 1$	$n_{2014} = 1$	$n_{2020} = 1$	$n_{2021} = 1$	$n_{2022} = 1$
$n_{2023} = 2$	$n_{2024} = 1$	$n_{2100} = 1$	$n_{2101} = 1$	$n_{2102} = 1$	$n_{2103} = 2$
$n_{2104} = 1$	$n_{2110} = 1$	$n_{2111} = 1$	$n_{2112} = 1$	$n_{2113} = 1$	$n_{2114} = 1$
$n_{2120} = 1$	$n_{2121} = 1$	$n_{2122} = 1$	$n_{2123} = 1$	$n_{2124} = 1$	$n_{2200} = 1$
$n_{2201} = 2$	$n_{2202} = 1$	$n_{2203} = 1$	$n_{2204} = 1$	$n_{2210} = 1$	$n_{2211} = 1$
$n_{2212} = 1$	$n_{2213} = 1$	$n_{2214} = 1$	$n_{2220} = 1$	$n_{2221} = 2$	$n_{2222} = 1$
$n_{2223} = 1$	$n_{2224} = 3$	$n_{2300} = 1$	$n_{2301} = 1$	$n_{2302} = 1$	$n_{2303} = 1$
$n_{2304} = 1$	$n_{2310} = 1$	$n_{2311} = 1$	$n_{2312} = 1$	$n_{2313} = 1$	$n_{2314} = 1$
$n_{2320} = 1$	$n_{2321} = 1$	$n_{2322} = 1$	$n_{2323} = 1$	$n_{2324} = 1$	$n_{2400} = 1$
$n_{2401} = 1$	$n_{2402} = 2$	$n_{2403} = 1$	$n_{2404} = 1$	$n_{2410} = 1$	$n_{2411} = 1$
$n_{2412} = 1$	$n_{2413} = 1$	$n_{2414} = 1$	$n_{2420} = 1$	$n_{2421} = 1$	$n_{2422} = 1$
$n_{2423} = 1$	$n_{2424} = 1$	$n_{3000} = 2$	$n_{3001} = 1$	$n_{3002} = 2$	$n_{3003} = 1$
$n_{3004} = 1$	$n_{3010} = 1$	$n_{3011} = 1$	$n_{3012} = 1$	$n_{3013} = 1$	$n_{3014} = 1$
$n_{3020} = 1$	$n_{3021} = 1$	$n_{3022} = 1$	$n_{3023} = 3$	$n_{3024} = 1$	$n_{3100} = 1$
$n_{3101} = 1$	$n_{3102} = 1$	$n_{3103} = 1$	$n_{3104} = 1$	$n_{3110} = 1$	$n_{3111} = 1$
$n_{3112} = 2$	$n_{3113} = 1$	$n_{3114} = 1$	$n_{3120} = 1$	$n_{3121} = 1$	$n_{3122} = 1$
$n_{3123} = 1$	$n_{3124} = 1$	$n_{3200} = 1$	$n_{3201} = 1$	$n_{3202} = 1$	$n_{3203} = 1$
$n_{3204} = 1$	$n_{3210} = 1$	$n_{3211} = 3$	$n_{3212} = 1$	$n_{3213} = 1$	$n_{3214} = 1$
$n_{3220} = 1$	$n_{3221} = 1$	$n_{3222} = 2$	$n_{3223} = 1$	$n_{3224} = 1$	$n_{3300} = 1$
$n_{3301} = 1$	$n_{3302} = 1$	$n_{3303} = 1$	$n_{3304} = 2$	$n_{3310} = 1$	$n_{3311} = 1$
$n_{3312} = 1$	$n_{3313} = 1$	$n_{3314} = 3$	$n_{3320} = 1$	$n_{3321} = 1$	$n_{3322} = 1$
$n_{3323} = 2$	$n_{3324} = 2$	$n_{3400} = 1$	$n_{3401} = 1$	$n_{3402} = 1$	$n_{3403} = 2$
$n_{3404} = 1$	$n_{3410} = 3$	$n_{3411} = 1$	$n_{3412} = 1$	$n_{3413} = 1$	$n_{3414} = 1$

Continued on next page

Table 1 – Continued from previous page

$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$
$n_{3420} = 1$	$n_{3421} = 1$	$n_{3422} = 1$	$n_{3423} = 1$	$n_{3424} = 1$	

$\sum_{j_1=0}^3 \sum_{j_2=0}^4 \sum_{j_3=0}^2 \sum_{j_4=0}^4 n_{j_1 j_2 j_3 j_4} = 345$ is got according to the above table, and the α -factor parameters can be easily obtained by Equation 7. For instance, $\alpha_{2314} = \frac{1}{345}$. Since there is no assumption that independence among the components, the joint probability distribution $P(f_1, f_2, f_3, f_4)$ is simply given by the α -factor parameters, which means $P(2, 3, 1, 4) = \alpha_{2314} = \frac{1}{345}$.

The survival signature of the complex system in Figure 1 is totally the same as calculated before as it only depends on the structure of the system. Combining the joint probability distribution values and their corresponding survival signature, the survival probability of the system after the CCfs $P(T_s > t \mid CCF)$, which can be calculated by Equation 6, is $\frac{461}{934}$. This value is still low, however, it has an improvement compared with the initial failure event in Case 1.

4.3. Case 3: general α -factor model 2

It can be seen from Case 2 that this system is highly susceptible to common cause failures, as all combinations of the failed components have occurred in the past data. There is a more robust system to be analysed in this Case, with past data can be seen in Table 2.

Table 2: Past data $n_{j_1 j_2 j_3 j_4}$ on the system in Figure 1

$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$	$n_{j_1 j_2 j_3 j_4}$
$n_{0001} = 16$	$n_{0002} = 11$	$n_{0003} = 13$	$n_{0004} = 6$	$n_{0010} = 13$	$n_{0011} = 3$
$n_{0012} = 2$	$n_{0013} = 3$	$n_{0014} = 3$	$n_{0020} = 13$	$n_{0021} = 5$	$n_{0022} = 2$
$n_{0023} = 4$	$n_{0100} = 13$	$n_{0101} = 3$	$n_{0102} = 3$	$n_{0103} = 2$	$n_{0104} = 2$
$n_{0110} = 2$	$n_{0111} = 1$	$n_{0120} = 3$	$n_{0200} = 13$	$n_{0201} = 4$	$n_{0202} = 2$
$n_{0203} = 1$	$n_{0204} = 13$	$n_{0210} = 1$	$n_{0220} = 3$	$n_{0300} = 14$	$n_{0301} = 5$
$n_{0302} = 2$	$n_{0303} = 13$	$n_{0304} = 1$	$n_{0310} = 3$	$n_{0320} = 2$	$n_{0400} = 7$
$n_{0401} = 3$	$n_{0402} = 3$	$n_{0403} = 2$	$n_{0404} = 1$	$n_{0410} = 1$	$n_{0420} = 3$
$n_{1000} = 17$	$n_{1001} = 6$	$n_{1002} = 3$	$n_{1003} = 4$	$n_{1004} = 4$	$n_{1010} = 5$
$n_{1011} = 1$	$n_{1020} = 3$	$n_{1100} = 6$	$n_{1101} = 2$	$n_{1102} = 1$	$n_{1110} = 2$
$n_{1111} = 1$	$n_{1200} = 3$	$n_{1201} = 1$	$n_{1300} = 6$	$n_{1301} = 1$	$n_{1400} = 4$
$n_{1401} = 1$	$n_{1402} = 1$	$n_{2000} = 13$	$n_{2001} = 5$	$n_{2002} = 3$	$n_{2003} = 2$
$n_{2004} = 2$	$n_{2010} = 4$	$n_{2011} = 1$	$n_{2020} = 4$	$n_{2100} = 6$	$n_{2101} = 1$
$n_{2110} = 1$	$n_{2200} = 3$	$n_{2300} = 3$	$n_{2301} = 1$	$n_{3000} = 9$	$n_{3001} = 5$
$n_{3002} = 3$	$n_{3010} = 4$	$n_{3020} = 2$	$n_{3100} = 3$	$n_{3300} = 1$	$n_{3301} = 1$

As can be seen from the above table that there is one past event (n_{1111}) that all component types are affected, and few past data where three types have been influenced ($n_{0111}, n_{1011}, n_{1101}, n_{1102}, n_{1110}, n_{1201}, n_{1401}, n_{1402}, n_{2011}, n_{2101}, n_{2110}, n_{2301}, n_{3301}$ in this case). In other words, this system has the feature that components belong to one type are more reliable when CCFs affect the other types. Therefore, it is unlikely to have large numbers of components of more than two types fail simultaneously.

Similarly, $\sum_{j_1=0}^3 \sum_{j_2=0}^4 \sum_{j_3=0}^2 \sum_{j_4=0}^4 n_{j_1 j_2 j_3 j_4} = 345$ and the joint probability distribution $P(f_1, f_2, f_3, f_4)$ can be got directly from α -factor parameters, with $P(f_1, f_2, f_3, f_4) = \alpha_{n_1 n_2 n_3 n_4}$. The survival signature of the system remains the same as before since its structure does not change at all.

Based on Equation 6, it can be known that $P(T_s > t \mid CCF) = \frac{439}{473}$, which means the system survival probability after a common cause failure is around 92.81%. This is highly improved compared with the former two cases.

4.4. Case 4: imprecision within system

In this subsection, let consider uncertainty in the system reliability analysis after common cause failures. In Case 1, if the system has not suffered CCF in the past, there might not be enough data to calculate the α -factor parameters, although the standard α -factor model can still be implemented by using experts' judgements.

Given that a total of 20 common cause component failures has occurred across the complex system in Figure 1, let two groups of experts estimate how the data would be spread. Suppose group one gives that $\alpha_1^1 = \frac{2}{5}$, $\alpha_2^1 = \frac{1}{5}$ and $\alpha_3^1 = \frac{2}{5}$ for components type 1, while for components type 2, $\alpha_1^2 = 0$, $\alpha_2^2 = \frac{2}{5}$, $\alpha_3^2 = \frac{2}{5}$ and $\alpha_4^2 = \frac{1}{5}$. $\alpha_1^3 = \frac{2}{3}$ and $\alpha_2^3 = \frac{1}{3}$ for components type 3, and for components type 4, $\alpha_1^4 = \frac{3}{7}$, $\alpha_2^4 = \frac{1}{7}$, $\alpha_3^4 = \frac{2}{7}$ and $\alpha_4^4 = \frac{1}{7}$. At this time, even the survival signature of $\Phi(2, 3, 2, 3)$ remains the same as $\frac{41}{48}$, which is a big value within the survival signature. However, its corresponding $P(1, 1, 1, 1)$ decreases from $\frac{1}{48}$ to 0. Let summarise the products of $P(f_1, f_2, f_3, f_4)$ and their corresponding $\Phi(m_1 - f_1, m_2 - f_2, m_3 - f_3, m_4 - f_4)$, then the survival probability $P(T_s > t \mid CCF)$ of the complex system after the next CCFs is $\frac{6}{23}$.

For group two, the estimated data are $\alpha_1^1 = \frac{1}{2}$, $\alpha_2^1 = \frac{1}{3}$, $\alpha_3^1 = \frac{1}{6}$, $\alpha_4^1 = \frac{1}{4}$, $\alpha_2^2 = \frac{1}{4}$, $\alpha_3^2 = \frac{1}{4}$, $\alpha_1^3 = \frac{1}{2}$, $\alpha_2^3 = \frac{1}{2}$, $\alpha_1^4 = \frac{3}{8}$, $\alpha_2^4 = \frac{3}{8}$, $\alpha_3^4 = \frac{1}{4}$ and $\alpha_4^4 = 0$. For this circumstance, $P(1, 1, 1, 1)$ increases to $\alpha_1^1 \alpha_1^2 \alpha_1^3 \alpha_1^4 = \frac{1}{2} * \frac{1}{4} * \frac{1}{2} * \frac{3}{8} = \frac{3}{128}$. Therefore, $P(T_s > t \mid CCF) = \frac{179}{456}$ according to Equation 6.

So due to the epistemic uncertainty in this example, the probability bounds of the system works after the next common cause failure event is $\bar{P}(T_s > t \mid CCF) = [\frac{6}{23}, \frac{3}{128}]$.

4.5. Results discussion

Common cause failures are the simultaneous failure of multiple similar components

due to the same root cause, these kind of failures have been shown to decrease the reliability and performance of complex systems.³³ A common cause failure event may affect an entire system or only a fraction of its components. Consequently, the number of components involved in a common cause failure event ranges from 1 to the total number of components within the same type.

In Case 1, due to the assumptions of the standard α -factor model, the survival probability of the system is unsatisfactorily low. In order to release these assumptions, it is necessary to generalise the α -factor model. Both Case 2 and Case 3 improve from $\frac{50}{139}$ to $\frac{461}{934}$ and $\frac{439}{473}$, respectively. The structure of the system remains the same for case 2 and case there, and the total number of the past data is identical to Case 2, but why the vulnerability of the system to CCFs declines so drastically? It can be investigated that the higher values of $P(f_1, f_2, f_3, f_4)$, $\frac{17}{345}$, $\frac{16}{345}$, $\frac{14}{345}$, $\frac{13}{345}$ and $\frac{11}{345}$, correspond to survival signature $\Phi(l_1, l_2, l_3, l_4) = 1$. Therefore, these kinds of CCFs that are triggered will not be likely to cause system fails. While in Case 2, there are more instances in the past data where larger numbers of components from more than two types had failed, making it less likely that the system could continue to work. Case 4 shows that epistemic uncertainty will lead the uncertainty within the system in this case. In order to reduce the imprecision, engineers need to put efforts to get the precise α -factor parameters.

5. Conclusions

Common cause failure events have highly adverse impact on the reliability and availability of systems in the real world, which makes it important to analyse systems reliability after CCFs. This paper put forwards the survival signature based reliability analysis on complex systems with common cause failures.

The survival signature is a summary of system structure function, which makes it is efficient to analyse complex systems. Based on the basic α -factor model, the paper presents the standard and general α -factor models for analysing complex system reliability with common cause failures. The effects of epistemic uncertainty are taken into account as well. What is more, the proposed methods are combined with the survival signature in order to perform complex system reliability analysis in the presence of common cause failure events. The feasibility and effectiveness of the proposed measures are demonstrated by the numerical examples.

The standard α -factor model has the assumptions that (1) the common cause failure event will affect all the common cause groups of components; and (2) the failure components are independent. All of which may lead to an unsatisfactory survival probability of system after a common cause failure event. In practice, the engineer will have to consider repairing or replacing the failed components if this kind of circumstance happens. Therefore, the CCFs model combines with the survival signature can be used to decide whether to repair or replace the failed components immediately, or after the next common cause failure event. In other words, the administrator or designer has to consider the cost of repairing or replace

the components as soon as possible, or taking the risk of allowing next common cause failure vent to occur before performing the failed components repair or replacement. Overall, it is an example of how decision theory is incorporated in the practical engineering world.

In order to release the assumptions of the former model, the general α -factor model is introduced. By investigating the two cases which are given for this model, it can be seen that design for reliability is useful in the engineering world. The goal of the reliability policy is to achieve high initial reliability by focusing on reliability fundamentals during design. So design for reliability provides engineers and managers with a range of tools and techniques for incorporating reliability into the design process for complex systems.³⁴ For complex system, it can examine the survival signature first, in order to find out which component combinations of failures will not allow the system to function at all. Therefore, the engineers can ensure these components are equipped to be more reliable under certain circumstances, which makes the system more reliable.

A possible challenge of the work presented in this paper is that there may not be enough past data on the realistic system, which is essential to calculate the precise values of the α -factor parameters. This is either if the system does not usually experience the common cause failures, or if the system has not been run in the past. However, these α -factor models can still be implemented by using experts' judgements on the system to ascertain the $n_{j_1, j_2, \dots, j_K}$ values.

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Declaration of Conflicting Interests

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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