Perpendicular diffusion of solar energetic particles: When is the diffusion approximation valid?

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Abstract. Multi-spacecraft observations of widespread solar energetic particle (SEP) events indicate that perpendicular (to the mean field) diffusion is an important SEP transport mechanism. However, this is in direct contrast to so-called spike and drop-out events, which indicate very little lateral transport. To better understand these seemingly incongruous observations, we discuss the recent progress made towards understanding and implementing perpendicular diffusion in transport models of SEP electrons. This includes a re-derivation of the relevant focused transport equation, a discussion surrounding the correct form of the pitch-angle dependent perpendicular diffusion coefficient and what turbulence quantities are needed as input, and how models lead to degenerate solutions of the particle intensity. Lastly, we evaluate the validity of a diffusion approach to SEP transport and conclude that it is valid when examining a large number of (an ensemble of) events, but that individual SEP events may exhibit coherent structures related to the magnetic field turbulence at short timescales that cannot be accounted for in this modelling approach.

1. Introduction

It is now established that widespread solar energetic particle (SEP) electron events can be explained by the presence of rather efficient perpendicular diffusion. However, there are still many outstanding issues such as e.g. the magnitude of the perpendicular diffusion coefficient and the nature of the turbulence leading to cross-field transport. In this proceeding we discuss the applicability of (diffusive) perpendicular diffusion of SEPs and how the inclusion of this process can lead to degenerate simulation results at 1 AU.

2. The (perpendicular) diffusion approximation

2.1. Transport equation

There is some criticism towards the addition of perpendicular diffusion to the focused transport equation, as it is usually derived without any cross-field transport terms [see e.g. 1]. However, Zhang [2] derives the focused transport equation including the effects of perpendicular transport by using the Vlaslov equation as a point of departure. Both the electric and magnetic field is written as the sum of a large scale average and a rapid fluctuating part. A coordinate transformation is then made to the wave frame (here assumed to be the solar wind frame) and another transformation to the particle's guiding center position, followed by a transformation from Cartesian to spherical coordinates in momentum space. The distribution function is also written as the sum of an ensemble average and a perturbation term and quasi-linear theory is then used as a closure for the perturbation part [see e.g. 1]. A similar approach is also outlined in le Roux and Webb [3]. This give rise to the diffusion terms and the perpendicular diffusion can be seen to arise from random guiding centre drifts. Lastly a gyrotropic distribution is assumed and an average over gyrophase is performed.

Alternatively, we can use the Fokker-Planck equation as a point of departure. A perturbation approach is not necessary because the Fokker-Planck coefficients already include the effects of random forces acting on the particles [see e.g. 4]. The same three transformations are again made, followed by a gyrophase average. Here, the perpendicular diffusion coefficients arise from a transformation of the Fokker-Planck coefficients to the guiding centre's position. This yields

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x_i}\left[\frac{dx_i}{dt}f\right] + \frac{\partial}{\partial p}\left[\frac{dp}{dt}f\right] + \frac{\partial}{\partial \mu}\left[\frac{d\mu}{dt}f\right] = \frac{\partial}{\partial \mu}\left[D_{\mu\mu}\frac{\partial f}{\partial \mu} + D^{\perp}_{\mu i}\frac{\partial f}{\partial x_i}\right] + \frac{\partial}{\partial x_i}\left[D^{\perp}_{i\mu}\frac{\partial f}{\partial \mu} + D^{\perp}_{ij}\frac{\partial f}{\partial x_j}\right], \tag{1}$$

where $D_{\mu\mu}$ is the pitch-angle diffusion coefficient and $D^{\perp}_{ij}$ are the perpendicular diffusion coefficients, with the mixed terms $D^{\perp}_{i\mu}$ usually neglected. Note that the distribution function $f(\vec{x}; p; \mu; t)$ is written in mixed coordinates, where the guiding center position $\vec{x}$ and time are measured in the observer's frame and the momentum p and pitch-cosine μ are measured in the SW frame. In Eq. 1,

$$\frac{d\vec{x}}{dt} = \mu v\, \hat{B}_0 + \vec{v}_{\rm sw} + \vec{v}_d \tag{2a}$$

$$\frac{dp}{dt} = p\left(\frac{1-3\mu^2}{2}\,\hat{B}_0\hat{B}_0 : \vec{\nabla}\vec{v}_{\rm sw} - \frac{1-\mu^2}{2}\vec{\nabla}\cdot\vec{v}_{\rm sw} - \frac{\mu}{v}\,\hat{B}_0\cdot\frac{d\vec{v}_{\rm sw}}{dt}\right) \tag{2b}$$

$$\frac{d\mu}{dt} = \frac{1-\mu^2}{2}\left(v\,\vec{\nabla}\cdot\hat{B}_0 + \mu\,\vec{\nabla}\cdot\vec{v}_{\rm sw} - 3\mu\,\hat{B}_0\hat{B}_0 : \vec{\nabla}\vec{v}_{\rm sw} - \frac{2}{v}\,\hat{B}_0\cdot\frac{d\vec{v}_{\rm sw}}{dt}\right), \tag{2c}$$

with v the particle speed, $\hat{B}_0 = \vec{B}_0/B_0$ a unit vector in the direction of the background magnetic field, $\vec{v}_{\rm sw}$ the solar wind velocity,

$$\vec{v}_d = \frac{\mu p}{q}\hat{B}_0 \times \frac{\partial}{\partial t}\left[\frac{\hat{B}_0}{B_0}\right] + \frac{p}{qvB_0}\frac{d\vec{v}_{\rm sw}}{dt}\times\hat{B}_0 + \frac{pv}{qB_0}\left[\mu^2\left(\vec{\nabla}\times\hat{B}_0\right)_{\perp} + \frac{1-\mu^2}{2}\frac{\hat{B}_0\times\vec{\nabla}B_0}{B_0}\right] \tag{3}$$

the gyrophase averaged guiding centre drift velocity perpendicular to the magnetic field [5, 6, 7], q the particle charge, and $\vec{a}\vec{b} : \vec{c}\vec{d} = a_i b_j c_j d_i$ a tensor contraction. The diffusion coefficients are also gyrophase averaged, although not explicitly indicated, and it is assumed that momentum diffusion is negligible in the solar wind. In the derivation it was additionally assumed that the solar wind is non-relativistic, and that $v \gg v_{\rm sw}$. See Skilling [8], Riffert [9], Ruffolo [10], Zhang [2], le Roux and Webb [11], le Roux et al. [12], and Zank [1] for details and discussions about the focused transport equation and its derivation.

This equation is usually implemented in a simplified way in SEP transport models. E.g. Strauss and Fichtner [13] neglects energy losses and drifts when considering SEP electrons. Several different approaches also exist to incorporate perpendicular diffusion, such as Laitinen et al. [14], while recent modelling studies [e.g. 15] also include drift effects.

The question is, however: *When is the diffusion approximation, for perpendicular diffusion, valid for solar energetic particles?*

2.2. Turbulent versus deterministic motion

Using the usual definition of turbulence, we can write,

$$\vec{B} = \vec{B}_0 + \vec{b}, \tag{4}$$

where $\vec{B}_0$ is the Parker [16] heliospheric magnetic field (HMF), and where the fluctuating components disappears after an *appropriate* averaging [e.g. 17],

$$\langle \vec{b} \rangle = 0. \tag{5}$$

Formally, the averaging is over an infinite ensemble of instantaneous field realizations, convected during $\Delta t \to \infty$ past the spacecraft (although, in principle, only a finite, but large, number of ensembles are needed, depending on the nature of the turbulence). The energetic particle population under consideration, however, determines what magnetic fluctuations constitute turbulence and what structures are deterministic in nature. The particle travels through only a single realization of the HMF, and only samples it for a (small) amount of time, at least for the case of SEPs, so that Δt becomes finite. For a distribution of particles starting with e.g. different gyro-phases, particles may follow field lines that rapidly diverge from each other so that $\langle \vec{b} \rangle \sim 0$ when averaging over all possible particle trajectories. If, on average, $\langle \vec{b} \rangle \sim 0$ does not hold, the field should, however, be decomposed as

$$\vec{B} = \vec{B}_0 + \delta\vec{B} + \vec{b}, \tag{6}$$

where $\langle \vec{b} \rangle = 0$ when averaging over the particle's propagation time (or equivalently averaging over the particle's propagation distance if homogeneous turbulence is considered), and $\delta\vec{B}$ represents deviations from the Parker HMF that the particle senses as coherent structures. Depending on the scale sizes, this could have very important consequences for particle transport: Usually, in transport models, we assume an unperturbed Parker HMF, with all the observed turbulence added to calculation of e.g. $D_\perp$. If, however, $\delta\vec{B} \neq 0$, we would need to add a perturbed Parker background HMF into a model (in which particles move deterministically) while the remaining turbulent fluctuations, $\vec{b}$, lead to perpendicular diffusion. One possible way to do this would be to simulate SEP transport on top of an MHD generated magnetic field, using photospheric/coronal observations to drive the internal boundary. An approach similar to e.g. Wijsen et al. [18]. The grid size of the MHD model could then be used as a scale separator, determining which magnetic perturbations are deterministic and which are turbulent (in this case the small scale fluctuations that cannot be captured in the MHD model due to their characteristic size being smaller than the numerical MHD grid resolution). On the other hand, this approach is not always satisfactory: Each instance of such a model, using a different photospheric map, will lead to a different magnetic field realization, and it becomes difficult (if not impossible) to make general predictions, related to the average behaviour of the SEPs.

The left panel of Fig. 1 shows a typically assumed 2D turbulence power spectrum, while the right panel shows the same graph, but plotted against a perpendicular length scale $l_\perp \sim k_\perp^{-1}$. The approximate position of the correlation length ($\langle l_\perp \rangle \sim 0.01$ AU) is indicated by the vertical green dashed line, while the vertical blue dashed line shows $l_\perp \sim 1$ AU, the approximate length a SEP travels in interplanetary space between its source at/near the Sun and an observer near Earth. It is clear that fluctuations with $l_\perp < \langle l_\perp \rangle$ will be sensed as turbulence by SEPs: A particle will sense (i.e. interact/move through) at least 100 of these structures before reaching Earth. A more detailed calculation by Laitinen and Dalla [19] suggest that the structures in the heliosphere are elongated along the mean magnetic field and thus the number of structures may be smaller.

However, what happens when fluctuations with scales of $l_\perp \sim 1$ AU are present? An SEP will only sense one such fluctuation during its propagation, and these structures would therefore be, as defined by the particle, a coherent structure. It could be re-stated that SEPs, for a given event in time, will only interact with a single ensemble (realization) of the turbulent magnetic field and for this case, the diffusion approximation would not be valid. However, we believe that there is, possibly, two ways to overcome this limitation: The first it to examine only large ensembles of SEP events, and a second would be if SEPs decouple efficiently from wandering field lines. Both processes are discussed further below.

2.3. Ensemble observations

Fig. 2 illustrates the idea behind different magnetic field ensembles (realizations): Red curves show examples of meandering field lines (field lines undergoing (diffusive) turbulent motion while being advected into space), while the blue symbols show the possible motion of a particle's guiding center, trapped on these field line without decoupling. For different times, different photospheric conditions and/or different random motions will result in different field realizations (ensembles). This would, in turn, lead to different particle distributions being observed away from the source as illustrated in the top panels of the figure. In this model, where the guiding center cannot decouple from the meandering field line, the distribution at the observer would the 'patchy', with particles only being observed if they are magnetically connected (via a turbulent field line) to the source. This is illustrated in e.g. the simulations of Tooprakai et al. [20]. Such a patchy distribution would be seen even if the magnetic field contains only very small scale turbulence: wandering field lines, from a small source region, advect from the Sun into interplanetary space, which is not empty, but already filled with existing magnetic field lines, while field lines originating within a small source can never fill the entire 1 AU volume as they must also be divergence free. In reality, the FLRW description of *random walking magnetic field lines* is only a first-order description of magnetic turbulence, and cannot capture any coherent MHD structures. This would mean that, in the example discussed above, a single SEP event will always only sample a single ensemble of the turbulent magnetic field. This in turn would mean that the SEP particles propagating along field lines without decoupling will never be in a diffusive state (as the magnetic field can never be in such a state).

However, we argue that if we consider a large ensemble of magnetic realizations, the average behaviour of the SEP will follow a field-line-random-walk-type [FLRW 21] description and in this limit a diffusive description for perpendicular transport will be appropriate. We therefore propose that, formally, a diffusive SEP model can never correctly describe the evolution of

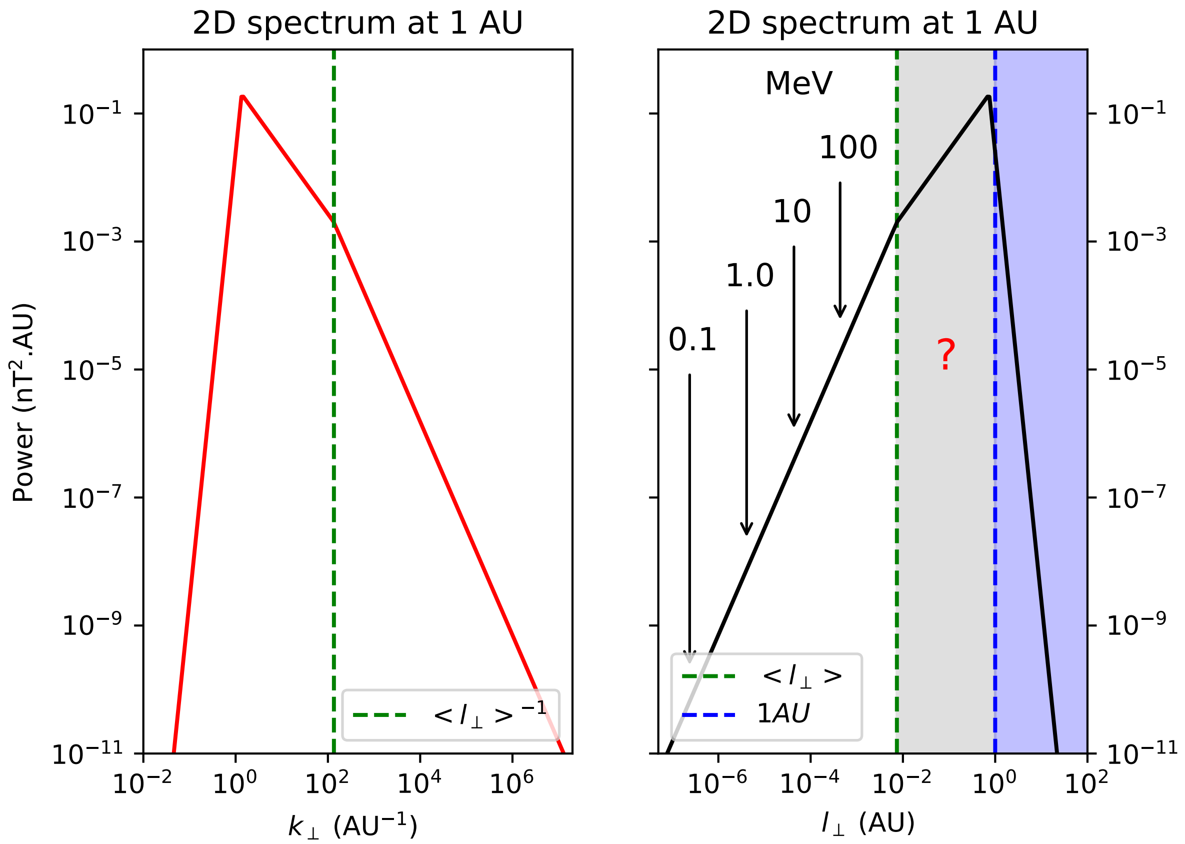

Figure 1. Left panel: An example of a typically assumed 2D turbulence spectrum. The green vertical dashed line shows the approximate correlation length. Right panel: The same turbulence spectrum, but now shown as a function of length rather than wavenumber. The blue vertical line indicates a length of 1 AU.

a single SEP event (if particle decoupling is negligible), but can correctly model the average behaviour of a large number of events. The simulations of e.g. Strauss et al. [22] would therefore be appropriate as these authors compare the results of their diffusive SEP model to a large set of ensemble observations. We also note that because a single SEP event will never show diffusive characteristics, while such characteristics are simulated by most SEP transport models, this could explain the dichotomy between so-called drop-out events (assumed to indicate very little lateral transport) and wide-spread events and numerical SEP transport models indicating that efficient lateral transport must be present: Drop-out events represents the SEP motion in a single magnetic field realization, while SEP transport models simulate the average behaviour and are unable to reproduce such small scale variations that result from a single magnetic field realization. However, these conclusions may change if particle decoupling (see the next section) is also considered. Note, that the problem of superluminal particle propagation related to the diffusion approximation (both in parallel and perpendicular transport) can in this context be explained as an artefact of the ensemble picture of transport, since for a single SEP event, particles do follow one representation of the potential field line configuration and can thus not propagate faster than the speed of light.

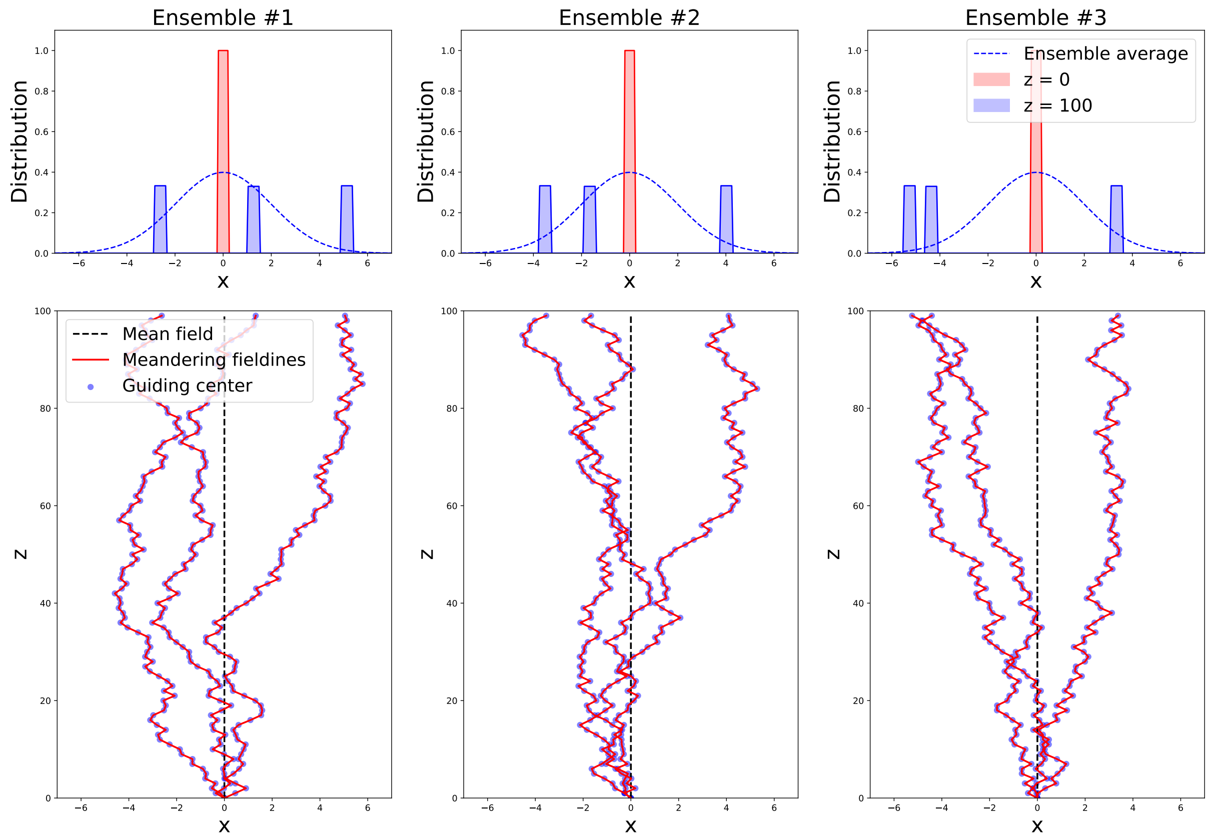

Figure 2. Illustration of how a particle's guiding center (blue dots) behaves in different turbulent magnetic field (red curves) ensembles/realizations. The top panels show the injected distribution ($z = 0$, red), the resulting particle distributions ($z = 100$, blue), and the prediction from a diffusion equation (dashed blue curve).

2.4. Decoupling from meandering field lines

The meandering field line description discussed in the previous section, and illustrated in Fig. 2 is, however, probably not complete. It is highly unlikely that SEPs, experiencing turbulent fluctuations of a range of different scales, will not decouple from a magnetic field. A useful way to describe the decoupling is to look at the guiding center drift in a turbulent magnetic field [see e.g. 23],

$$ {}_gV_y \approx v \left\{ \overbrace{\mu \frac{b_y}{B_0}}^{\text{streaming}} + \overbrace{\sqrt{\Gamma}\frac{V_A}{v}\frac{b_y}{B_0}}^{\text{E-field}} + \overbrace{\frac{1}{2}\frac{r_L}{l_x}(1-\mu^2)\left(\frac{b_\perp^2}{B_0^2} + \frac{b_z}{B_0}\right)}^{\text{gradient}} + \overbrace{\frac{r_L \mu^2}{l_z}\frac{b_x}{B_0}}^{\text{curvature}} \right\} \hat{y}, \tag{7} $$

for the y-component of the drift speed in a 2D turbulence geometry where

$$ \vec{b} \cdot \vec{b} = b_x^2 + b_y^2 + b_z^2 = b_\perp^2 + b_z^2. \tag{8} $$

The time a particle takes to decouple from its original field line can then be approximated as

$$ \tau_{\text{dec}} = \frac{\langle l_y \rangle}{\langle {}_gV_y^2 \rangle^{1/2}}, \tag{9} $$

where $\langle l_y \rangle$ is the characteristic (i.e. average) turbulence length scale.

Using such a simplified approach, and assuming an exponential time decorrelation, Strauss et al. [23] showed that the perpendicular diffusion coefficient can be approximated as

$$D_\perp \approx \overbrace{v\langle l_\perp \rangle |\mu| \frac{\langle b^2 \rangle}{B_0}^{1/2}}^{\text{FLRW}} + \overbrace{v r_L (1-\mu^2) \frac{\langle b^2 \rangle}{B_0^2}}^{\text{Gradients}} + \overbrace{\sqrt{\Gamma} V_A \langle l_\perp \rangle \frac{\langle b^2 \rangle^{1/2}}{B_0}}^{\text{E-field}}, \tag{10}$$

where the first term is simply particle motion along the fluctuating field, and the second and third terms represent perpendicular motion (i.e. decoupling) due to random gradients and electric fields. For the present calculation we do not consider perpendicular *scattering* of SEPs due to e.g. propagating wavemodes. Such a discussion can be found in e.g. le Roux and Webb [3]. It is interesting to note that the curvature in Eq. 7 does not contribute to diffusion in this model as $l_z \to \infty$ in a purely 2D geometry.

Fig. 3 illustrates the process of guiding center decoupling, becoming increasingly efficient towards the right panel. We also note that if particles efficiently decouple from the meandering field lines, and thereby fill the initially empty flux tubes with particles, the perpendicular spreading could again be represented by a diffusion process, even for a single magnetic field realization: By decoupling from their original field lines, particle sample different large scale fluctuations, and thereby sample different magnetic field ensembles.

2.5. Diffusion coefficients

Eq. 10 can connect the FLRW to a more detailed theory by using the fact that [see also 24]

$$\langle l_\perp \rangle^2 \sim \langle k_\perp^{-2} \rangle = \frac{\int k_\perp^{-2} g^{2\text{D}}(k_\perp) dk_\perp}{\int g^{2\text{D}}(k_\perp) dk_\perp} \tag{11}$$

is the so-called ultrascale of turbulence [25], which then leads to [see Eq. (3.42) of 26]

$$\kappa_{FL}^2 = \langle l_\perp \rangle^2 \frac{\delta B_{2\text{D}}^2}{B_0^2} = \frac{2\pi}{B_0^2} \int_0^\infty k_\perp^{-2} g^{2\text{D}}(k_\perp) dk_\perp. \tag{12}$$

Then, in the limit of $r_L \ll \langle l_\perp \rangle$, where only the first term is Eq. 10 dominates, we have

$$D_\perp^{\text{FLRW}}(r_L \ll \langle l_\perp \rangle) = a v |\mu| \kappa_{\text{FL}}, \tag{13}$$

where a was introduced following Qin and Shalchi [27]. This allows us to write the complete perpendicular diffusion coefficient as

$$D_\perp \approx \left(a v |\mu| + \sqrt{\Gamma} V_A \right) \kappa_{FL} + v r_L (1-\mu^2) \frac{\langle b^2 \rangle}{B_0^2}, \tag{14}$$

where

$$\langle b^2 \rangle = \delta B_{2D}^2 = 2\pi \int_0^\infty g^{2\text{D}}(k_\perp) dk_\perp. \tag{15}$$

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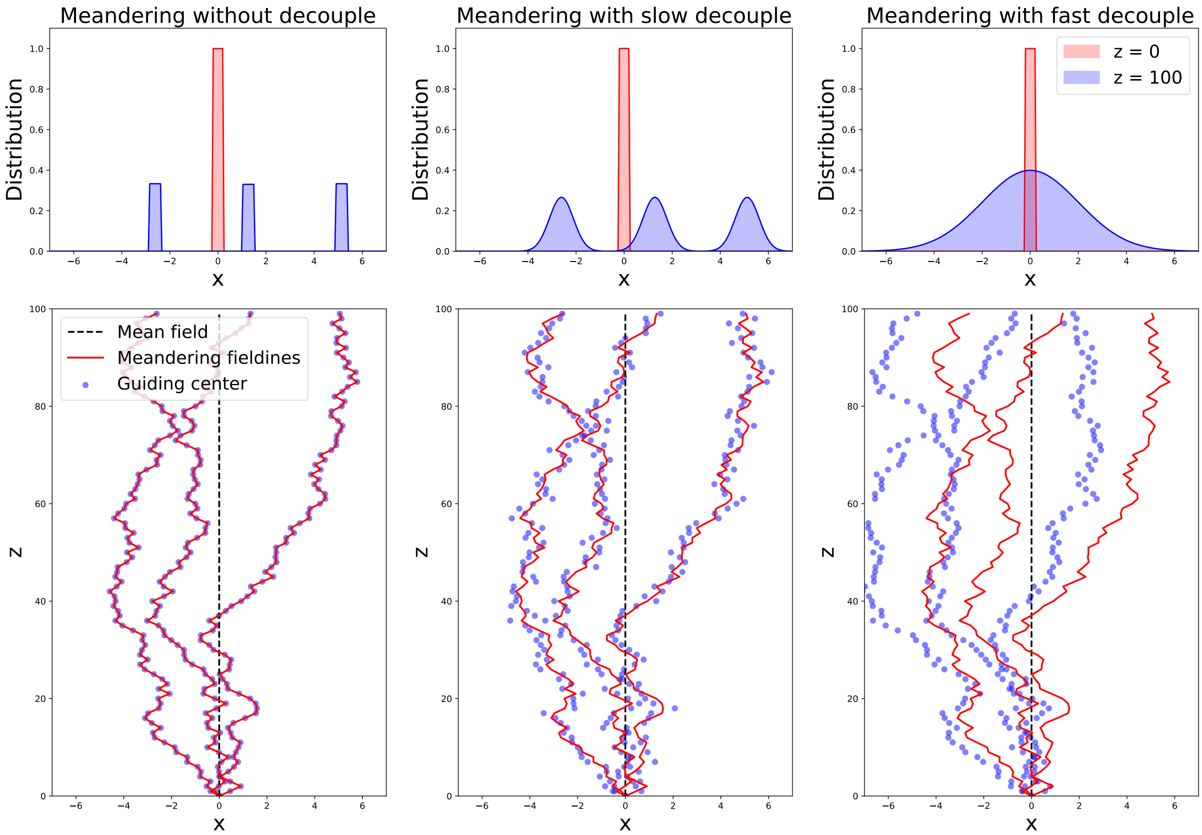

Figure 3. Similar to Fig. 2, but for a single turbulent realization, but with different levels of decoupling.

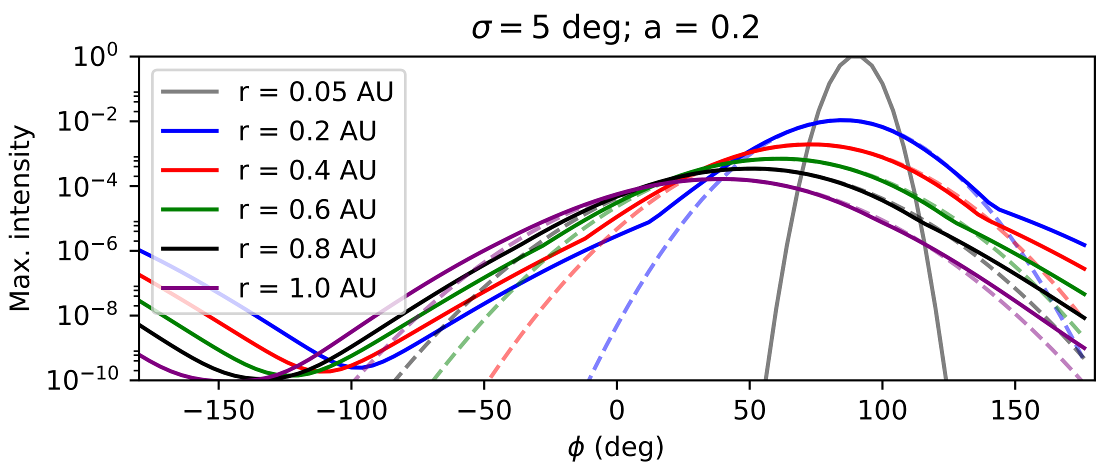

Figure 4. The solid lines show the TOM intensity, as a function of longitude, for different radial positions. The dashed lines are fitted Gaussian distributions.

3. Degeneracy associated with simulation results

Lastly, we discuss the fact that models, assuming diffusive perpendicular diffusion as a dominant transport process, can lead to degenerate results at e.g. Earth. Using the model of Strauss

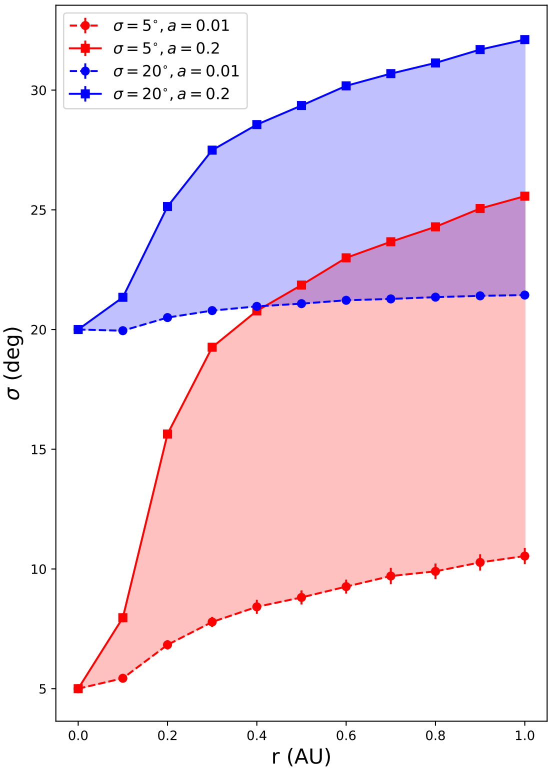

Figure 5. The broadness of the fitted Gaussian distribution, as a function of radial distance, for different levels of perpendicular diffusion and different injection source widths.

et al. [22] we perform simulations using a Gaussian source with a broadness of $\sigma = 5^\circ$ and perpendicular diffusion with efficiency parametrized by $a = 0.2$, Fig. 4 show the resulting time-of-maximum (TOM) intensities, as a function of longitude, for different radial positions. At each position a Gaussian distribution is fitted to the results, and the broadness thereof determined (fits are shown in the figure by dashed curves). This is repeated for different source widths and different levels of perpendicular diffusion, and the results are shown in Fig. 5. The figure shows that a combination of different source widths and levels of perpendicular diffusion can lead to a similar distribution at Earth. This is also the main reason for the ongoing debate regarding the source of widespread SEP event: Are these events cause by a large (i.e. broad) source, or due to efficient cross-field transport? Of course, as Fig. 5 shows, this degeneracy disappears when we get closer to the Sun so that *Solar Orbiter* and/or *Parker Solar Probe* might be able to easily disentangle the source vs transport processes. Additionally, more realistic magnetic field geometries (i.e. non-Parkerian fields), might not show this degeneracy so clearly.

A different approach is to also look at other observables, e.g. the anisotropy of the widespread event and the onset time as a function of magnetic connectivity. Dresing et al. [28] present such simultaneous observations, while the simulations of Strauss et al. [22] shows how these combined observations can be compared to model results. These authors conclude that some level of perpendicular diffusion must always be present in a numerical model to reproduce the observed quantities.

4. Summary and Conclusions

In this proceeding we have shown that:

- The perpendicular diffusion term arises naturally in the focused transport equation if drift motion in the turbulent magnetic field is considered.

- As the turbulent fluctuations, observed at 1 AU, contain fluctuation with length scales up to $\sim$ 1 AU, the diffusion approximation is, most likely, not applicable when simulating individual SEP events. However, we propose that this approximation can still be applied to ensemble observations. The diffusive description may still be the best (and perhaps the only) approach for SEP forecasting applications since one can never know the exact meandering field line topology in advance, due to it's stochastic nature.
- The diffusive perpendicular diffusion approximation would be more appropriate if particles can decorrelate efficiently from wandering magnetic field lines. It is, however, unclear how efficient this decoupling process is. Laitinen and Dalla [29] present simulations where particle decouple (very) slowly from meandering field lines, while Laitinen et al. [30] also find that initial particle propagation is consistent with ballistic motion along meandering field lines. In addition, Chollet and Giacalone [31] find very little decoupling when examining so-called drop-out events.
- When considering only simulations of the differential intensity, we show that simulation results are degenerate, with the degeneracy disappearing when moving closer to the source.

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